ELSEVIER

Available online at www.sciencedirect.com

SCIENCE DIRECT®

Optics Communications 244 (2005) 389–398

OPTICS COMMUNICATIONS

www.elsevier.com/locate/optcom

Fourier factorization of nonlinear Maxwell equations in periodic media: application to the optical Kerr effect

Nicolas Bonod *, Evgueni Popov, Michel Nevière

Institut Fresnel, case 161, UMR 6133, Université de Provence, Faculté des Sciences et Techniques de St-Jérôme,
CLARTE, 13397 Marseille Cedex 20, France

Received 28 May 2004; received in revised form 6 September 2004; accepted 7 September 2004

Abstract

The recently developed fast Fourier factorization method resolves linear Maxwell equations in a truncated Fourier basis using correct factorization rules. In nonlinear optics, Maxwell equations present a discontinuous product of two simultaneously discontinuous functions for which no rule of factorization applies. Using an iterative method which avoids such a type of factorization, we extend the fast Fourier factorization method to nonlinear optics. We demonstrate the good convergence of the method by studying deep metallic gratings with grooves filled with a nonlinear material, illuminated in TM polarization with a high intensity plane wave.

PACS: 42.79.D; 42.65
Keywords: Diffraction gratings optical; Nonlinear optics

---

1. Introduction

Numerical analysis of periodic media using the differential method requires the projection of Maxwell equations on a truncated basis. It is now well known that the method of factorization of a product of two functions in a truncated Fourier basis [1,2] and more generally on any truncated basis of continuous functions (e.g. Bessel functions [3]), depends on the continuity of the functions and their product. The Laurent's rule gives the Fourier components of a product of two functions in an infinite Fourier basis. As soon as the Fourier basis is truncated, the Laurent's rule, then called "direct rule" presents a bad convergence with respect to the number of Fourier components involved, when the two functions of the product are simultaneously discontinuous. If their product is continuous, it exists another rule, called "inverse rule", which gives

---

[*] Corresponding author. Tel.: +33 491288824; fax: +33 491674428.
E-mail address: nicolas.bonod@fresnel.fr (N. Bonod).

0030-4018/$ - see front matter © 2004 Elsevier B.V. All rights reserved.
doi:10.1016/j.optcom.2004.09.026

rapidly converging results. Established in 1996 in a truncated Fourier basis by Li [1] and recently extended in any truncated basis of continuous functions [3], this rule enabled a new formulation of the differential theory of diffraction by periodic media [4]. The new method, called "fast Fourier factorization" (FFF) [2,4] has drastically improved the convergence of the differential method when applied to metallic gratings in TM polarization [4]. However, when the two functions and their product are simultaneously discontinuous, no rule of factorization could enable a good convergence [1], so that the results given by both the direct and the inverse rule have a poor convergence. This is the case in nonlinear optics, when gratings consisting of optically nonlinear materials are considered. In that case, the vector nonlinear polarization can be expressed as the product of a tensor of nonlinear susceptibility by the electric field vector taken at a given power. At the grating surface, the three last functions are simultaneously discontinuous. In that case, the representation of the Fourier components of the product in terms of the two other function Fourier components, using direct or inverse rule is poorly converging. Thus, the factorization of the discontinuous product of discontinuous functions has to be avoided. This paper proposes a method to deal with this problem. The nonlinear problem is treated through an iterative process with respect to the nonlinearity, solving a linear diffraction problem at each iteration step. In the linear diffraction problem, the use of the fast Fourier factorization method allows for the spatial reconstruction of the electric field. The determination of the dielectric permittivity for the next iteration step, is done at any point of the coordinate space from the spatial distribution of the electric field and the nonlinear susceptibility, the latter being zero outside the nonlinear domain and constant inside it. The next iteration resolves a linear diffraction problem with a new spatial distribution of the dielectric permittivity and the fast Fourier transform (FFT) algorithm is then used to find its Fourier components. Such a process, i.e. the calculation of the Fourier components of the dielectric permittivity from its reconstruction in the coordinate space avoids the factorization of a product of type (3) enounced by Li, namely a discontinuous product of two discontinuous functions.

The paper is structured in the following manner. In Section 2, we present the nonlinear Maxwell equations (Section 2.1) from which we deduce the relation between the nonlinear dielectric permittivity and the electric field. Once this relation obtained, we present the iterative method with respect to the nonlinear dielectric permittivity (Section 2.2). We use the FFF method to calculate at each step the Fourier components of the electric field using the direct and the inverse rule (Section 2.3). We have then to calculate for the next iteration the Fourier components of the nonlinear dielectric permittivity.

Once the theoretical method is presented, we show numerical experiments on deep metallic gratings made of nonlinear media illuminated in TM polarization (Section 3.1). Such a configuration requires the use of the FFF method in linear optics. First, we show the convergence of the presented method as a function of the number of Fourier components as it was done in linear optics with the FFF method and the classical differential method (Section 3.2). Then, we study the convergence of the method with respect to the number of iterations (Section 3.3). Then, once the convergence of the method is proved, we study nonlinear effects in metallic gratings illuminated by a plane wave with a high amplitude to obtain large nonlinear effects (Section 3.4).

2. Resolution of nonlinear Maxwell equations using the factorization rules in a truncated Fourier basis

2.1. Maxwell equations in nonlinear optics

Formally, Maxwell equations are the same in linear and nonlinear optic

$$\mathrm{curl}\mathbf{E}(\mathbf{r},t) = -\frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t}, \tag{1}$$

$$\mathrm{curl}\mathbf{H}(\mathbf{r},t) = \frac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t}, \tag{2}$$

but a nonlinearity will arise from the behaviour of the medium. We consider in this study media with-