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An efficient organization mechanism for spatial networks

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Abstract

Spatial networks, also known as random geometric graphs, are random graphs with certain distance metric, in which each node is connected to some others within its neighborhood disc. Due to the rapid increase of network scales, the design of spatial networks becomes increasingly challenging. Inspired by the recently discovered small-world topology in relational networks, we identify an efficient organization mechanism for spatial networks, which we believe is useful for spatial network design. Two types of discs are introduced. Edges in the large discs are counterparts of "shortcuts." We find that such "two-radius" spatial networks exhibit small characteristic path length, yet with low cost. This mechanism is applied to broadcasting protocol design for wireless ad hoc/sensor networks.

Keywords: Statistical physics of complex networks; Spatial network design; Small world; Optimization; Wireless ad hoc/sensor networks

1. Introduction

Popular network models can be classified as relational and spatial networks. The constructions of relational networks do not depend upon any external metric of distance [1], while spatial networks are networks with certain metric, such as Euclidean distance [2]. The design of networks becomes more and more challenging than before. Global optimization seems impossible due to the rapid increase of the network scales. The success of existing large-scale man-made networks depends heavily on the decentralizing nature of the rules in their organization mechanisms. The entire network should be efficient both globally and locally. It should be efficient on a global scale as its function is usually realized by the joint coordination among many nodes. Its measurement is the typical separation between two vertices in the graph. Meanwhile, the network should be efficient on a local scale. The measurements include the clustering/cliquishness of a local neighborhood (first introduced in Ref. [5]), and the cost to build a neighborhood (first introduced in Ref. [4]). It is found in Refs. [3–5] that, to design an efficient organization mechanism for both relational and spatial networks, small degrees of separation, high clustering, and low cost are all required.

One of the most important discoveries for decentralized organization mechanism for large-scale networks is the small-world topology. By analogy with the small-world phenomenon in social networks, Watts and Strogatz [5] have shown that many real relational networks possess small-world characteristics, that is, small

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typical separation between two vertices (measured by characteristic path length) and high clustering (measured by clustering coefficient). Note that the Watts–Strogatz model assumes the sparseness of the edges.

Whether the small worlds still emerge in spatial networks? On the one hand, as Watts analyzed in Ref. [6], spatial networks admit no small-world class, unless some edges have long ranges (shortcuts) that are comparable to the network size. On the other hand, according to Refs. [7,8], after relaxing the strict definition of spatial networks, i.e., affecting the disc-like neighborhoods by adding ‘‘shortcuts’’ with probabilities depending on inter-node distance, small worlds can be introduced. Based on these two observations, it is natural to ask, when the definition is yet strictly obeyed, whether spatial networks present similar behaviors like the small worlds. In fact, obeying this definition is meaningful in both theoretical (e.g. relational vs. spatial networks) and practical (e.g. wireless systems with omni antenna) perspectives.

For spatial networks, in reality, the clustering tends to be consistently high due to locality of the edges [11,12]. Meanwhile, motivated by applications, we regard the node operational cost as another indispensable local metric for network design. In wireless applications, the cost turns out to be the power consumption of wireless devices, and the fact that the battery power is not unlimited should be taken into account. In this paper, we are focusing on whether there exist some efficient organization mechanisms for spatial networks so that they have both small typical separation and low cost (measured by Node Degree, as defined later. It is a kind of measure for cost or energy consumption in wireless applications, as will be explained in Section 5).

Note the conclusion of Watts on spatial networks was made based on the use of the clustering coefficient as the only efficiency measurement on a local scale. If we also take into account the node degree that measures the number of neighbors of a typical node in the graph, the key concept of shortcuts may still be applicable for spatial networks. This paper shows that in this sense a counterpart of shortcuts does exist for spatial networks. Observe that if we connect a node to all neighbors in a large neighborhood disc centered at this node in a random geometric network, then the appearance of such discs provides shortcuts to reduce the characteristic path length efficiently. Observe also that such large discs increase the cost, namely increase the node degree. The new efficient organization mechanisms we find for spatial networks are based on the above two observations and can be regarded as an extension of the small-world topology for relational networks. The idea is to construct random geometric networks by connecting most of the nodes with ‘‘nonshortcuts’’ and adding ‘‘shortcuts’’ connections randomly with small probability. The ‘‘nonshortcuts’’ connections are corresponding to edges in small discs, which result with large separation and low cost; and the ‘‘shortcuts’’ are corresponding to edges in large discs. Because there are two types of discs in the networks we call them ‘‘two-radius’’ spatial networks. We will show in this paper they exhibit small separation, high clustering, and low cost.

The rest of this paper is organized as follows. In Section 2, we propose three spatial network models. Based on these models, in Section 3, collective dynamics of the two-radius spatial networks are presented. Then we provide analysis from the perspective of optimization in Section 4, and apply our results to wireless ad hoc/sensor networks in Section 5. Section 6 concludes the paper.

2. Three spatial network models

A spatial network is a graph where each node (vertex) is assigned random coordinates in a d-dimensional box of unit volume, and each edge is added to connect pairs of nodes which are close to each other by metric. Here 2-dimensional spatial networks with Euclidean distances are considered. In the square area $[0,1] \times [0,1]$, N nodes are assigned random coordinates (x_i, y_i) $(i = 1, 2, \ldots, N)$, thus with density $\rho = N$. Note that no periodic boundary conditions are used here.

We assume that the neighborhood ranges of all nodes are circles, that is, each node only ‘‘communicate’’ with some others within its neighborhood disc. Let r_i denote the disc radius of node i. For any other node j, if $|(x_j, y_j) - (x_i, y_i)| < r_i$ is true, it defines a directed link $i \to j$, and j is then i's outgoing neighbor. If the link $j \to i$ also exists, then the two nodes are connected via a bidirectional link $i \leftrightarrow j$ and are able to directly arrive back and forth. In the following part, to explore efficient behaviors in spatial networks, three models are presented: model I, uniform small radius; model II, uniform large radius; and model III, two-radius model, viz. small/large radii.