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# Adaptive Lévy walks can outperform composite Brownian walks in non-destructive random searching scenarios

A.M. Reynolds *

*Rothamsted Research, West Common, Harpenden, Hertfordshire AL5 2JQ, UK*

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ABSTRACT

Recently it has been found that composite Brownian walk searches are more efficient than any Lévy walk when searching is non-destructive and when the Lévy walks are not responsive to conditions found in the search. Here a new class of adaptive Lévy walk searches is presented that encompasses composite Brownian walks as a special case. In these new models, bouts of Lévy walk searching alternate with bouts of more intensive Brownian walk searching. Switching from extensive to intensive searching is prompted by the detection of a target. And here, switching back to extensive searching arises if a target is not located after travelling a distance equal to the 'giving-up distance'. It is found that adaptive Lévy walks outperform composite Brownian walks when searching for sparsely distributed resources. Consequently there is stronger selection pressures for Lévy processes when resources are sparsely distributed within unpredictable environments. The findings reconcile Lévy walk search theory with the ubiquity of two modes of searching by predators and with their switching search mode immediately after finding a prey.


## 1. Introduction

Over recent years there has been an accumulation of evidence from a variety of experimental, theoretical and field studies that many organisms use a movement strategy approximated by Lévy walks when they are searching for resources [1–6]. Lévy walks are comprised of random sequences of straight-line movements with lengths, $l$, drawn from a probability distribution function having a power-law tail, $p(l) \sim l^{-\mu}$ where $1 < \mu < 3$. Lévy walks have no characteristic scale because the variance of $p(l)$ is a divergent quantity. Lévy walks are therefore said to be 'scale-free'. Qualitatively a Lévy walk is characterised by frequently occurring but relatively short movements that are punctuated by more rarely occurring longer movements, which in turn are punctuated by even rarer, even longer movements. Over much iteration, a Lévy walk will be distributed much farther from its starting position than a Gaussian (Brownian) random walk of the same length. The scale-free and super-diffusive properties of Lévy walks can lead to advantages over Gaussian motions in random search scenarios [7]. Recently, however, the optimality of Lévy walk searches has been called into question by Benhamou [8] and by Plank and James [9] who have suggested that composite Brownian walks are more efficient than Lévy walks when searching for sparsely and patchily distributed resources (i.e. where individual targets occur in spatially localised clusters). Additionally, they showed that it may be very difficult, in practice, to distinguish between composite Brownian walks and Lévy walks. This led them to question whether some apparent instances of Lévy movement patterns can really be attributed to underlying Lévy processes. This is significant because the key to understanding and prediction lies in the elucidation of processes underlying observed movement patterns [10].

* Tel.: +44 0 1582 763133; fax: +44 0 1582 760981.
*E-mail address:* andy.reynolds@bbsrc.ac.uk.

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Benhamou [8] devised his composite Brownian walk model for the location of patchily distributed targets that once visited become temporally unavailable either because they have become depleted or because of the increased risk of predation. In this model searchers travel out from the origin of their search in a straight line until they encounter a target and then proceed to search destructively within the patch that contains this target using Brownian movements. If a target is not located within a prescribed time then the searcher switches back to the original straight-line motion. Benhamou [8] showed that his composite Brownian walk model is more efficient than any Lévy walk that is not responsive to conditions found in the search. Reynolds [11] subsequently pointed out that the composite Brownian walk model of Benhamou [8] can, in fact, be interpreted as being an 'adaptive' or responsive Lévy walk search. This correspondence arises because straight-line movements between targets correspond to truncated $\mu \to 1$ Lévy walks whilst Brownian walks correspond to Lévy-like movements with $\mu \geq 3$. Benhamou [8] has therefore demonstrated that an adaptive Lévy walk is better than any non-adaptive Lévy walk when searching destructively in patchy environments. Moreover, this composite Brownian walk is entirely consistent with standard Lévy walk search theory which predicts that straight-line movements are optimal when searching destructively for sparsely distributed patches whilst Brownian movements are optimal for within-patch searching [7,11].

More recently Plank and James [9] found that the searching efficiency of a composite Brownian walk was higher than that of any non-adaptive Lévy walk when searching non-destructively for patchily distributed resources. In a non-destructive search, targets are not depleted once visited but instead remain targets for future searches. As in the composite Brownian walk model of Benhamou [8], the key ingredients of this model are straight-line searches interspersed with more intensive Brownian searches. This model is at variance with the expectation from Lévy walk search theory that $\mu = 2$ Lévy walks rather than straight-line movements are optimal when searching non-destructively for sparsely distributed patches.

Here, a new class of adaptive Lévy walk searching model is presented that effectively encompasses the model of Benhamou [8], and that of Plank and James [9] as a special case (attained when $\mu \to 1$). It is shown that optimal adaptive Lévy walks are characterised by $\mu > 1$ and that they can outperform composite Brownian walks when searching non-destructively for patchily distributed resources.

## 2. Adaptive Lévy walks

The adaptive Lévy walk search model developed here is a straightforward extension of the classic Lévy walk search model devised by Viswanathan et al. [7]. A searcher travels out from the origin of the search with movements prescribed by the Lévy walk search model of Viswanathan et al. [7]. When in this extensive mode of searching, a searcher moves in a straight line towards the nearest target if the target lies within a 'direct vision' distance, $r$; otherwise the searcher chooses a direction at random and a distance, $l$, drawn from a power-law distribution, $P(l) = (\mu - 1)\, r^{\mu-1} l^{-\mu}$ for $l \geq r$ and $P(l) = 0$ for $l < r$. It then moves incrementally towards the new location whilst constantly seeking for targets within a radius, $r$. If no target is detected, it stops after traversing the distance $l$ and chooses a new direction and a new distance; otherwise it proceeds to the target. Detection of a target triggers a change from Lévy walk movements to more intensive Brownian movements. This switching does not arise in classic Lévy walk searches where instead detection of a target triggers the truncation of the current Lévy walk and the start of a new Lévy walk search from a nearby location (perhaps mimicking the cumulative effects of an intensive searching phase). In the intensive searching phase the searcher moves in a straight line towards the nearest target if the target is within a 'direct vision' distance; otherwise the searcher chooses a direction at random and moves a distance, $r$. If no target is detected, it stops after traversing a distance $r$ and chooses a new direction; otherwise it proceeds to the target. During the intensive search, the searcher continually monitors the distance travelled since last locating a target. If a target is not detected after travelling a distance equal to the 'giving-up distance', $l_{GU}$, then the searcher abandons the intensive search and resumes the Lévy walk search.

This adaptive Lévy walk search model effectively reduces to the composite Brownian walk model of Benhamou [8], and that of Plank and James [9] when $\mu \to 1$, and here this adaptive Lévy walk is used as a surrogate for the model of Plank and James [9] in the execution of numerical simulations.

## 3. Searching efficiencies

Attention is focused on one-dimensional searches for the location of targets that are located within regularly spaced patches, with spacing $\lambda$. Model predictions do not change qualitatively when the patches are randomly distributed in space with mean spacing $\lambda$. The results from the analysis of one-dimensional searches are expected to carry over to two and three dimensions, as noted by Plank and James [9]. In the numerical simulations a patch is represented by a single target. Once such a target has been detected, the searcher is displaced by a distance $10r$ before resuming its search. As for Plank and James [9], this scenario represents a searcher in a patchy environment with only limited but reliable knowledge of its surroundings, and is very similar to the non-destructive scenario of Viswanathan et al. [7]. If the searcher moves in the right direction, it will remain within the patch and quickly locate the next target. If it moves in the other direction, it will move out of the patch and potentially have to travel a long distance to the next target. Model predictions for the relative merits of adaptive and non-adaptive Lévy walks, and composite Brownian walks do not depend sensitively on the displacement distance.

Here following Viswanathan et al. [7], the searching efficiency of a model is taken to be the reciprocal of the mean distance travelled before encountering a target. In other words, it is the mean number of targets located during a search divided by