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Numerical irreversibility in self-gravitating small N-body systems, II: Influence of instability affected by softening parameters

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ABSTRACT

We investigate the fundamental characteristics of numerical irreversibility appearing in self-gravitating small N-body systems by means of a molecular dynamics method from the viewpoint of time-reversible dynamics. We reconsider a closed spherical system consisting of 250 point-particles interacting through the Plummer softened potential. To investigate the characteristics of numerical irreversibility, we examine the influence of the instability affected by the softening parameter for the softened potential (the instability considered here is the instability of a dynamical system in chaos theory, e.g., a separation rate of the distance between two nearby trajectories in phase space or speed space). To this end, under the restriction of constant initial energy, the softening parameter for the Plummer softened potential is varied from $0.005R$ to $0.050R$, where R is the radius of the spherical container. We first confirm that the size of the softening parameter, i.e., the deviation of the potential from a pure gravitational potential, influences the virial ratio, the density, the pressure on the spherical container, etc., during an early stage of the relaxation process. Through a time-reversible simulation based on a velocity inversion technique, we demonstrate that numerical irreversibility due to round-off errors appears more rapidly with decreasing softening parameter. This means that the higher the instability of the system or the higher the separation rate of the distance between two nearby trajectories, the earlier the memory of the initial conditions is lost. We show that the memory loss time t_m, when the simulated trajectory completely forgets its initial conditions, increases approximately linearly with the timescale of the chaotic system, i.e., the Lyapunov time t_λ. In a small self-gravitating system, propagation of numerical irreversibility or loss of reversibility depends on both the energy state of the system and the instability affected by the softening parameter.

1. Introduction

Stellar self-gravitating N-body systems have attracted considerable research attention due to the fact that they exhibit properties quite different from short-range interacting systems, e.g., negative specific heat, violent relaxation and non-equilibrium nonextensive statistical mechanics [1–7]. Since N-body problems cannot be solved analytically, numerical simulations are important for studying N-body systems [8,9]. For example, in 1963, Aarseth simulated dynamical evolution of clusters of galaxies using the Plummer softened potential $\sim -1/(r^2 + r_0^2)^{1/2}$, where r_0 and r represent the softening parameter and the distance between particles, respectively [10]. However, it is known that numerical simulations inherently include round-off errors and that numerical irreversibility arises from the round-off errors, even in time-reversible systems.

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In the 1960s, Miller [11,12] examined irreversibility in small stellar dynamical systems and showed that numerical errors grow exponentially with time. Subsequently, many researchers, e.g., Lecar [13], Standish [14], Gurzadyan et al. [15], Kandrup [16], have investigated self-gravitating systems from the viewpoint of instability [17–29]. The instability discussed by them is the instability of a dynamical system in chaos theory, e.g., a separation rate of the distance between two nearby trajectories in phase space. However, irreversibility appearing in self-gravitating N-body systems has not yet been clarified from the viewpoint of time-reversible dynamics. In fact, even in short-range interacting systems [30–32], the influence of round-off errors or numerical irreversibility has not been investigated quantitatively, except for a few simple models [33–35] or for the works by the present authors [36–38].

Accordingly, to bridge the gap between short-range and long-range interacting systems, we investigated the numerical irreversibility appearing in self-gravitating N-body systems through a time-reversible simulation [39]. In a previous paper (Physica A 387 (2008) 2267; hereafter Paper I), we examined the influence of energy states and the integration step size Δt. In particular, in Paper I, to investigate the fundamental characteristics of numerical irreversibility, the softening parameter r_0 for the Plummer softened potential was fixed, since r_0 affects the instability of the system. That is, we have not examined the influence of the instability affected by the softening parameter on the numerical irreversibility. However, we can expect that the instability of a system, as suggested by Krylov [40], is closely related to irreversibility or loss of reversibility [31].

In most self-gravitating N-body simulations, in order to avoid noisy force estimates due to close encounters between particles or to simulate collisionless systems, the true potential $\sim -1/r$ is replaced by an artificially softened potential, e.g., $\sim -1/(r^2 + r_0^2)^{1/2}$. The influence of such softening parameters on N-body simulations has been investigated by Standish [14], Suto [19] and many other researchers [41–46]. In particular, the influence of the softening parameter on instability was examined in detail by Kandrup et al. [21] and Goodman et al. [22]. Recently, optimal softening for N-body simulations has been examined and discussed in an effort to minimize average errors in force calculations [47–51]. However, numerical irreversibility or loss of reversibility has not been investigated from the viewpoint of time-reversible dynamics. Moreover, although the instability affected by the softening parameter has been generally studied using phase trajectories, a relationship between the instability and numerical irreversibility has not been clear and not been investigated quantitatively. This is because the instability of the system isn't irreversibility itself, even if they could be closely related to each other. Therefore, we can expect that it is worthwhile to study numerical irreversibility and to examine the relationship between them quantitatively.

In this context, to clarify the relationship between irreversibility and instability affected by the softening parameter, we investigate numerical irreversibility appearing in a self-gravitating system through a time-reversible simulation. We consider a system consisting of N point-particles enclosed in a spherical container of radius R with reflecting walls [52–57]. To simulate an unstable system, the softening parameter r_0 is varied between $0.005R$ and $0.050R$, under a restriction of constant initial energy. That is, we examine the influence of the instability affected by the softening parameter on numerical irreversibility for a deeper understanding of simulations of self-gravitating N-body systems; i.e., through the simulation, we investigate numerical irreversibility appearing in those unstable systems.

The present paper is organized as follows: In Section 2, we give a brief review of numerical techniques for simulating a self-gravitating system enclosed in a spherical container with a reflecting wall. We describe the initial conditions for the simulation and a velocity inversion technique for time-reversible simulations. We also define several parameters for observing the unstable and irreversible behavior of a system. In Section 3, the simulation results are presented. Through a typical relaxation process, we first examine the influence of the softening parameter in Section 3.1. In Section 3.2, based on the time-reversible simulation, the relationship between the numerical irreversibility and instability of the system is investigated and discussed. In Section 3.3, the influence of the time step or the integration step size Δt is examined. Finally, we present our conclusions.

2. Methods

We consider a system consisting of N point-particles enclosed in a spherical container of radius R with reflecting (adiabatic) walls, i.e., the Antonov problem [1]. Although a method to simulate the Antonov problem has been described in Paper I, the details in the present simulation are slightly different. Therefore, in this section, we briefly review the present method.

2.1. Numerical models

To simulate a self-gravitating system, we integrate the set of classical equations of motion for the particles interacting through the Plummer softened potential. The Plummer softened potential Φ is given by

$$\Phi = -\frac{1}{\sqrt{r^2 + r_0^2}}, \tag{1}$$

where r and r_0 represent the distance between particles and the softening parameter, respectively. It is well-known that variations in the softening parameter r_0 influence the instability of a self-gravitating system [19–24]. Accordingly, in order to