Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
Physica A 363 (2006) 299–306
PHYSICA A
www.elsevier.com/locate/physa

Corrections to scaling and probability distribution of avalanches for the stochastic Zhang sandpile model

Duan-Ming Zhang[a], Yan-Ping Yin[a,*], Gui-Jun Pan[a,b], Fan Sun[a]

[a] *Department of Physics, Huazhong University of Science and Technology, Wuhan 430074, China*
[b] *Faculty of Physics and Electronic Technology , Hubei University, Whan 430062, China*

Received 12 May 2005
Available online 9 September 2005

Abstract

We study the distributions of dissipative and nondissipative avalanches separately in the stochastic Zhang (SP-Z) sandpile in two dimension. We find that dissipative and nondissipative avalanches obey simple power laws and do not have the logarithmic correction, while the avalanche distributions in the Abelian Manna model should include a logarithmic correction. We use the moment analysis to determine the numerical critical exponents of dissipative and nondissipative avalanches, respectively, and find that they are different from the corresponding values in the Abelian Manna model. All these indicate that the stochastic Zhang model and the Abelian Manna model belong to distinct universality classes, which imply that the Abelian symmetry breaking changes the scaling behavior of the avalanches in the case of the stochastic sandpile model.

Keywords: Self-organized criticality; Sandpile model; Dissipative; Nondissipative; Moment analysis

1. Introduction

Since its introduction in 1987, the sandpile model has been considered as the prototype of a self-organized critical (SOC) system [1]. The identification of the universality classes is the most important problem in the field of SOC. It still remains an unclear problem that whether the Abelian symmetry breaking changes the scaling behavior of the avalanches in the stochastic sandpile model. Recent studies showed that the non-Abelian stochastic directed sandpile model (NA-SDM) and the Abelian stochastic directed sandpile model (A-SDM) belong to the same universality class under the condition of parallel update [2]. However, our previous work on directed stochastic models presented that the NA-SDM and the A-SDM do not belong to the same universality class [3]. It seems to imply that the Abelian symmetry breaking changes the scaling behavior of the avalanches in the directed stochastic sandpile model. In the case of stochastic sandpile model, does the Abelian symmetry breaking change the scaling behavior of the avalanches? In order to get a deeper

*Corresponding author. Tel./fax: +86 027 87543882.
E-mail address: Aegeanfish@eyou.com (Y.-P. Yin).

0378-4371/$ - see front matter
doi:10.1016/j.physa.2005.08.032

understanding of this problem, we reconsider the distributions of dissipative and nondissipative avalanches in the stochastic Zhang model.

Drossel performed that in two-dimensional BTW sandpile, dissipative avalanche distributions follow clean power laws [4]. Distributions of nondissipative avalanches must also follow power laws in the infinite-size limit, but are subject to much stronger corrections to scaling. Dickman and Campelo showed that in one and two-dimensional (2D) Manna's sandpile, avalanche distributions in general do not follow simple power laws, but rather include a logarithmic correction [5]. Thus , it would be interesting to find whether the logarithmic correction appears in other models exhibiting SOC.

After the Zhang model was introduced by Zhang [6] in 1989, many numerical works were made on this model [7–9]. In this paper we study the dissipative and nondissipative avalanches separately in the two-dimensional stochastic Zhang (SP-Z) model. We find that the distributions of dissipative and nondissipative avalanches of the stochastic Zhang model exhibit a standard FSS behavior, and follow pure power laws that they do not have the logarithmic correction to scaling. It seems to imply that the logarithmic correction presented by Ref. [5] is dependent on the inherent mechanism of the Abelian Manna model. We also determine the critical exponents of dissipative and nondissipative avalanches by using the moment analysis [10–14]. Our principal result is that the scaling behavior of dissipative and nondissipative avalanches in the stochastic Zhang sandpile is different from that in the Abelian Manna model. It indicates that the stochastic Zhang model and the Abelian Manna model belong to different universality classes.

2. Model and simulations

The SP-Z model [9] is defined on a D-dimensional square lattice of linear size L in which we assign a nonnegative continuous variable E_i called "energy" on each site. At each time step, an amount of energy δ is added to a randomly chosen site j according to $E_j \to E_j + \delta$. The quantity δ is a random variable uniformly distributed in $[0, \delta_{\max}]$. In our simulations we consider the fixed value $\delta_{\max} = 0.25$. When a site acquires an energy larger than or equal to 1 ($E_i \geqslant 1$), it becomes active and topples. An active site i relaxes losing all its energy, $E_i \to 0$, which is randomly redistributed among its nearest neighbors. In the practical implementation of this rule, we draw four random numbers $\varepsilon_{i'}$, $0 \leqslant \varepsilon_{i'} \leqslant 1$, with $\sum_{i'} \varepsilon_{i'} = 1$ and update the nearest neighbors i' by $E_{i'} \to E_{i'} + \varepsilon_{i'} E_i$. We study the parallel updating that all active sites release their energy simultaneously.

We analyze the dissipative and nondissipative avalanches separately. Dissipative avalanches are those in which some energy leaves the system, while nondissipative avalanches are those in which no energy leaves the system. We reported three systems of different sizes for $L = 160$, 320, and 640 in two dimension. Our results are based on samples of about 10^7 avalanches for all the systems.

We present the avalanche distributions $P_s(s)$ of avalanche sizes s while "size" means the number of topplings in an avalanche, as shown in Fig. 1. The size distributions of dissipative avalanches are presented in Fig. 1(a), while the size distributions of nondissipative avalanches are shown in Fig. 1(b). The morphology of these two kinds of avalanche distributions generally includes a plateau-like region for small s, a rapidly decaying portion for large s, and a power-law-like interval between these limiting regimes. The power-law interval is increased with the size of the system. The probability distribution in the second and third portion generally follows:

$$P_s(s) = s^{-\tau_s} f_s(s/s_c) , \tag{1}$$

where f_s is a cutoff function decaying rapidly for large argument. The cutoff function f_s must take a constant value for $s \prec s_c$. And s_c is the cutoff characteristic size which diverges as $s_c \sim L^{\beta_s}$ when the system size L goes to infinity.

We compare distributions of dissipative avalanches with those of nondissipative avalanches for system of $L = 640$ in Fig. 2. It is illustrated that the two kinds of avalanche distributions obey the power laws but have different slopes of the power-law-like interval, which indicates that they present two distinct scaling behaviors. In other words, whether the avalanche is dissipative or nondissipative does not alter the power-law distributions of avalanches, but it changes the scaling behavior of avalanches.

Dickman and Campelo showed that distributions of dissipative and nondissipative avalanches in Abelian Manna's sandpile do not follow pure power laws [5], but include a logarithmic correction to scaling