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Novel order parameter to describe the critical behavior of Ising spin glass models

F. Romá[a], F. Nieto[a], A.J. Ramirez-Pastor[a,*], E.E. Vogel[b]

[a]Departamento de Física, Universidad Nacional de San Luis - CONICET, Chacabuco 917, 5700 San Luis, Argentina
[b]Departamento de Física, Universidad de La Frontera, Casilla 54-D, Temuco, Chile

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Abstract

A novel order parameter Φ for spin glasses is defined based on topological criteria and with a clear physical interpretation. Φ is first investigated for well known magnetic systems and then applied to the Edwards–Anderson $\pm J$ model on a square lattice, comparing its properties with the usual q order parameter. Finite size scaling procedures are performed. Results and analyses based on Φ confirm a zero temperature phase transition and allow to identify the low temperature phase. The advantages of Φ are brought out and its physical meaning is established.

Keywords: Lattice theory and statistics; Spin-glass and other random models; Phase transitions

1. Introduction

Despite over three decades of intensive work, the nature of the low temperature phase of two-dimensional Edwards–Anderson (EA) [1] model for spin glasses remains controversial. It is agreed that a phase transition occurs at zero temperature for a Gaussian distribution of bonds (GD) [2–5]. Similarly, for a symmetric $\pm J$ distribution or bimodal distribution (BD) of bonds, very convincing numerical evidence has been found that there is no transition at finite temperature [5–11]. In most of these references, the authors do not use an order parameter for characterizing the phase transition. On the other hand, data arising from other contributions, which are based on the behavior of a standard overlapping order parameter, support the existence of a finite critical temperature [12–14].

In this context, the main purposes of this paper are the following: (a) to show that the disagreement pointed out in previous paragraph is related to the non-zero overlap of site-order parameters obtained for quite distinct energy valleys; (b) to overcome this situation by proposing here a novel order parameter Φ, which is quite drastic to characterize phases but still is general enough to coincide with usual descriptions of ferromagnetic (F) and antiferromagnetic (AF) systems; (c) to apply Φ to do a scaling analysis for

*Corresponding author. Fax +54 2652 430224.
E-mail addresses: froma@unsl.edu.ar (F. Romá), fnieto@unsl.edu.ar (F. Nieto), antorami@unsl.edu.ar (A.J. Ramirez-Pastor), ee_vogel@ufro.cl (E.E. Vogel).

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two-dimensional EA systems including Binder cumulant [15]; (d) to confirm the assumption of the zero-temperature phase transition for two-dimensional BD, thus reinforcing this result obtained by previously quoted authors; and (e) to give a physical meaning to this finding by using the grounds on which Φ is based on.

The present work is organized as it follows. In Section 2, we introduce the model and define a novel order parameter, Φ, very useful for spin glasses and other frustrated systems. Results of the simulation are presented in Section 3. Finally, our conclusions are drawn in Section 4.

2. Model and basic definitions

Let us begin by very briefly introducing the system under study. Ising spin s_i occupies ith site of a two dimensional (square for simplicity) lattice. The interaction with the spin at site j is mediated by exchange interaction J_{ij}. In the absence of magnetic field (which is the case for the scope of the present paper) the Hamiltonian of such system can then be written as

$$H = \sum_{\langle i,j \rangle} J_{ij} s_i s_j \ , \tag{1}$$

where interactions $\{J_{ij}\}$ are restricted to nearest neighbor couplings. In the F Ising model, $J_{ij} = -J \ \forall \langle i,j \rangle$. For the EA model, we will consider half of the bonds F, while the other half will be described by AF bonds of the same magnitude, namely, $J_{ij} = +J \ (J > 0)$. A sample is one of the possible random distributions of these mixed bonds. For simplicity spins take values $s_j = \pm 1$, which can be equally denoted by their signs.

Now, let us consider a configuration α defined by a collection of ordered spin orientations $\{s_j^\alpha\}$. The usual EA order parameter q is built up by means of overlaps between two configurations α and β and takes the form

$$q_{\alpha\beta} = \frac{1}{N} \sum_{j=1}^{N} s_j^\alpha s_j^\beta \ , \tag{2}$$

where $N \ (\equiv L \times L)$ is the total number of spins.

For models in which the ground state is non degenerate after breaking ergodicity, such as the pure F case, the distribution of $q_{\alpha\beta}$ values for the ground manifold ($T = 0$) is trivial and it is given by delta functions at $q_{\alpha\beta} = 1.0$ and $q_{\alpha\beta} = -1.0$. This also happens in general for all systems with non-degenerate ground level. But this also applies to GD, where local fields have all different values at different sites, leading to a true minimum energy for just one pair of opposite ground states. However, for the BD the local field assumes a few discrete values only, which necessarily means highly degenerate ground manifolds leading to $|q_{\alpha\beta}| < 1.0$, for a large number of possible pairs of ground states. This distribution will have two broad symmetric maxima but it will not vanish in the intermediate region [13].

On the other hand, a more detailed description based on a topological picture of the ground state of BD was presented [16,17]. This framework allows us to define a state function with a clear physical meaning, which is a good candidate to be a new order parameter for a phase transition. In fact, it has been reported an important feature of the ground state, namely, at $T = 0$ there exist clusters of solidary spins (CSS) preserving the magnetic memory of the system (solidary spins maintain their relative orientation for all states of the ground manifold) [18]. The main idea of this work is to characterize the nature of the low temperature phase through the CSS.

Let us consider a particular sample of any given size N. We denote by Γ_κ any of the n CSS of the sample (κ runs from 1 to n). Calculations begin recognizing all of the CSS of each sample belonging to a set of 2000 randomly generated samples of each size. This process is closely related to finding the so-called "diluted lattice" that prevails after removing all frustrated bonds [19], so the algorithms designed for that purpose can also be used here.

Let us first pick any arbitrary ground state configuration denoted by an asterisk (∗) fixing one of the possible relative orientations of the CSS, thus becoming a reference configuration. Then a local overlap