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A new macro model with consideration of the traffic interruption probability

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ABSTRACT

In this paper, we present a new macro model which involves the effects that the probability of traffic interruption has on the car-following behavior through formulating the inner relationship between micro and macro variables. Linear stability analysis shows that consideration of the traffic interruption probability can improve the stability of traffic flow if and only if the drivers' reactive time required for adjusting their acceleration based on the traffic interruption probability p is not greater than that one based on the non-interruption probability $1 - p$. Numerical results verify that the new model can be used to analyze the effects of traffic interruption probability and traffic interruption on shock, rarefaction wave, small perturbation and uniform flow. The model has been applied in reproducing some complex traffic phenomena resulted by some traffic interruptions (*e.g.*, signal light, pedestrian and tolling station).

1. Introduction

Many traffic flow models have been developed to describe the physical mechanisms of various complex traffic phenomena [1–16] (*e.g.*, the formation mechanism and the propagating properties of various traffic waves and jams, the mechanisms of lane-changing and overtaking, the interactions among multiple vehicles). However, the existing models do not involve the effects of traffic interruption probability on traffic flow, and then can not directly be used to study the complex traffic phenomena resulted by various traffic interruptions. In fact, some traffic interruptions (*e.g.*, accidents) always occur with some probabilities and produce complex phenomena. Wong et al. [17–19] studied the contributing factors to traffic accidents. Telesca and Lovallo [20] analyzed the temporal properties in traffic accident time series and found that the time dynamics of traffic accidents is not Possonian but long-range correlated with periodicities ranging from 12 h to one year. Recently, Baykal-Gürsoy et al. [21] used the queuing theory to model traffic flow interrupted by incidents. The models proposed in Refs. [17–21] can reproduce some traffic phenomena resulted by accidents, but can not be used to evaluate the effects of various traffic interruption factors on the dynamic properties of traffic flow, since the traffic interruption probability is not considered explicitly.

In this paper, we first analyze the effects that the probability of traffic interruption has on the car-following behavior. Based on the inner relationship between micro and macro variables [22], we develop a new macro model with consideration of the traffic interruption probability. Then, we numerically investigate the effects that the traffic interruption probability

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and one traffic interruption (*e.g*., accident) have on the stability of traffic flow, shock, rarefaction wave, small perturbation and uniform flow. Finally, we apply our model to analyze the complex traffic phenomena caused by some traffic interruptions (*e.g*., signal light, pedestrian and tolling station).

2. The car-following model

In general, the single-lane car-following model can be written as follows [1]:

$$\frac{\mathrm{d}^2 x_n}{\mathrm{d}t^2} = f\left(v_n, \Delta x_n, \Delta v_n\right), \tag{1}$$

where f is the stimulus function, x_n and v_n are the position and speed of the nth vehicle respectively, $\Delta x_n = x_{n+1} - x_n$ is the headway, $\Delta v_n = v_{n+1} - v_n$ is the relative speed. Eq. (1) states that the acceleration of a vehicle is determined by the speed v_n, the headway Δx_n and the relative speed Δv_n. In order to improve the stability of traffic flow, scholars later developed some improved car-following models as follows [23–27]:

$$\frac{\mathrm{d}^2 x_n}{\mathrm{d}t^2} = f\left(v_n, \Delta x_n, \Delta x_{n+1}, \ldots, \Delta x_{n+m}, \Delta v_n\right), \tag{2}$$

where $\Delta x_{n+i} = x_{n+i+1} - x_{n+i}$. Zhao and Gao [28] found that a collision will occur under certain given conditions when using the FVD (full velocity difference) model [29] to describe traffic flow. They then proposed a new model with consideration of the effects that the acceleration of the leading vehicle has on the following vehicle, *i.e*.,

$$\frac{\mathrm{d}^2 x_n}{\mathrm{d}t^2} = f\left(v_n, \Delta x_n, \Delta v_n, \mathrm{d}^2 x_{n+1}/\mathrm{d}t^2\right). \tag{3}$$

To further enhance the stability of traffic flow, Wang et al. [30] proposed a multiple speed difference model as follows:

$$\frac{\mathrm{d}^2 x_n}{\mathrm{d}t^2} = f\left(v_n, \Delta x_n, \Delta v_n, \Delta v_{n+1}, \ldots, \Delta v_{n+k}\right), \tag{4}$$

where $\Delta v_{n+i} = v_{n+i+1} - v_{n+i}$.

The above car-following models can describe some complex phenomena, but can not directly be used to study the phenomena resulted by some traffic interruption factors. In fact, each vehicle may be interrupted with some probability. Considering this, we rewrite the acceleration equation of the nth vehicle, as follows:

$$\frac{\mathrm{d}v_n(t)}{\mathrm{d}t} = \kappa\left(V\left(\Delta x_n\right) - v_n\right) + \lambda_1 p_{n+1}\left(-v_n\right) + \lambda_2\left(1 - p_{n+1}\right)\Delta v_n, \tag{5}$$

where p_{n+1} is the probability that the leading vehicle is interrupted, κ, λ_1 and λ_2 are the reactive coefficients, and $V\left(\Delta x_n\left(t\right)\right)$ is the optimal speed of the nth vehicle at time t. Once the leading vehicle is completely interrupted, its speed immediately becomes zero, *i.e*., the speed difference between the $(n+1)$th and the nth vehicles takes $(-v_n)$. Eq. (5) states that the acceleration of the nth vehicle is determined by the speed v_n, the headway Δx_n, the relative speed Δv_n and the probability p_{n+1}.

3. The macro model

In order to study the effects that the probability of traffic interruption has on the dynamic properties of traffic flow, we should transform the micro variables in Eq. (5) into the macro ones by using the method of Ref. [22], *i.e*.,

$$\begin{aligned} &V\left(\Delta x_n(t)\right) \to v_e(\rho), \qquad v_n(t) \to v(x,t), \qquad v_{n+1}(t) \to v(x+\Delta,t), \\ &\kappa \to \frac{1}{T}, \qquad \lambda_1 \to \frac{1}{\tau_1}, \qquad \lambda_2 \to \frac{1}{\tau_2}, \qquad p_{n+1} \to p(x,t). \end{aligned} \tag{6}$$

The parameter Δ in $(x+\Delta)$ is the distance between the leading and following vehicles. The details of the inner relationship between macro and micro variables can be found in Refs. [6,9,22]. Then, Eq. (5) can be rewritten as follows:

$$\frac{\mathrm{d}v(x,t)}{\mathrm{d}t} = \frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = \frac{v_e\left(\rho\right) - v}{T} + \frac{1}{\tau_1}p\left(-v\right) + \frac{1}{\tau_2}\left(1 - p\right)\left(v\left(x+\Delta,t\right) - v\left(x,t\right)\right), \tag{7}$$

where v_e, v and ρ are respectively the equilibrium speed, the speed and the density. T, τ_1 and τ_2 are respectively the three reactive times required by drivers for adjusting their acceleration based on equilibrium speed, the speed difference $(-v)$ caused by traffic interruption and the speed difference $v\left(x+\Delta,t\right) - v\left(x,t\right)$ without traffic interruption. In general, $\kappa > \lambda_2$ and $\tau_1 < \tau_2$ hold, we then have $\tau_1 < \tau_2 < T$.