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Stochastic resonance and Brownian ratchets

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Abstract

We discuss the connections between Brownian ratchets (BR) and stochastic resonance (SR). We consider a periodic potential energy landscape with no left–right symmetry that is driven by an external force which can be derived from a potential that is periodic both in time and space. We show that this system presents two thermal enhancements within two different windows of the temperature. One is associated with a "coherent diffusion" by which particles jump back and forth between the minima of the periodic potential in synchrony with the external driving. The other is instead associated with a "coherent directional transport" by which particles hop synchronically from one minimum of the ratchet to the next. We calculate the current and the diffusion coefficients and show how transport undergoes a resonant enhancement. While the former is always present, the second only appears when left–right symmetry is broken.

Keywords: Transport phenomena; Diffusion; Brownian motors; Coherent transport; Thermal enhancement

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1. Introduction

Brownian Ratchets (BR) are devices out of equilibrium in which fluctuations make possible the directional transport of particles along a periodic potential with some left–right asymmetry. These devices that where first proposed by Smoluchowsky [1] and later discussed by Feynamnn [2] have deserved a great deal of attention in the literature (for reviews see Refs. [3,4]). There is a wide diversity of areas in which BRs are applied, for instance the working of molecular motors [5], the description of ion channels and molecular transport within cells [6] and the treatment of Parrondo's paradoxical games [7–9].

Stochastic resonance (SR) also represents a physical situation in which fluctuations play a similarly constructive role. It consists of a noise assisted enhancement by which power from the whole noise spectrum is pumped into a single mode that is coherent with an external driving force. This was first proposed in Ref. [10] to explain long term fluctuations in the Earth's climate but has later triggered a vast field of research [11] in which the biophysics of neural systems has a particularly important role [12–14]. In Ref. [15] the theory of SR has been discussed for two state systems and in Ref. [16], it is presented as a synchronization of the hopping mechanism between wells induced by the external periodic driving. In Refs. [17,18] a more general treatment of this theory is explained. A review of SR can be found in Ref. [19].

Although BR and SR are different physical phenomena, the fact that both can be placed within the same realm of noise-assisted non-equilibrium phenomena have induced several authors [6,20,21] to state that BR and SR may share some common underlying physical concepts. An attempt of a connection between SR and BR has been reported in Ref. [22] for a BR based on a Parrondo paradoxical game. The interference of two games plays the role of a random perturbation that gives rise to a resonant enhancement of the profit.

The present paper aims at bridging the gap between the theories of SR and BR by discussing how both phenomena may occur in the same physical system in spite of having deep physical differences. We do this by presenting a particular case of a BR in which the directional transport of particles can *resonantly* be enhanced within a window of values for the temperature. We refer to this device as a *stochastic resonant ratchet* (SRR). This model helps to discuss the similarities and the differences between the physics underlying BRs and SR. On the one hand the SRR can be assimilated to a spatially extended SR: particles are subject to thermal fluctuations and are placed in a periodic potential. A weak, periodic, external driving causes consecutive wells of the ratchet to alternate as absolute minima. Different levels of thermal noise are therefore expected to give rise to various regimes for the hopping of particles between consecutive minima, and thus to the transport of particles along the potential landscape. On the other hand, within the general theory of BR [4], the SRR can be framed as a *tilting ratchet* bearing also some similarity to the particular case of a *rocking ratchet* presented in Ref. [24], with a different type of external driving term.

The present work is organized as follows: in Section 2 we introduce the model under study. In Section 3 we discuss how this system presents two SRs at two