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Dynamical volatilities for yen–dollar exchange rates

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Abstract

We study the continuous time random walk theory from financial tick data of the yen–dollar exchange rate transacted at the Japanese financial market. The dynamical behavior of returns and volatilities in this case is particularly treated at the long-time limit. We find that the volatility for prices shows a power-law with anomalous scaling exponents $\kappa = 0.92$ (1 min) and 0.78 (10 min) and that our behavior occurs in the subdiffusive process. Our result presented will be compared with that of recent numerical calculations.

Keywords: Continuous time random walk; Returns; Volatility; Scaling exponent

1. Introduction

The investigation of the continuous time random walk (CTRW) theory between economists and physicists has recently received considerable attention as one interdisciplinary field [1,2]. This subject has particularly led to a better understanding

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for novel universal properties based on statistical concepts and methods. Particularly, one of prominent problems for the regular and disordered systems is the random walk theory [3] in the stochastic process. This theory has extensively developed to the CTRW theory, formerly introduced by Montroll and Weiss [4], which is essentially characterized both by the transition probability dependent of the length between steps and by the distribution of the pausing times [5]. Until now, the CTRW theory has even more extended to study in natural, applied, and social sciences, and among many outstanding topics, there have been mainly concentrated on the reactive and strange kinetics [6,7], fractional diffusion equations [8], random networks, earthquake modeling, hydrology, and financial options [9].

Scalas [10] has recently presented the correlation function for bond walks from the time series of bond and BTP (Buoni del tesoro Poliennali) futures exchanged at the London International Financial Futures and Options Exchange (LIFFE). Scalas et al. [11] have discussed that CTRW theory is applied to the dynamical behavior for tick-by-tick data in financial markets. Mainardi et al. [12] have discussed the waiting-time distribution for bond futures traded at LIFFE. Kim and Yoon [13] investigated the dynamical behavior of volume tick data for the bond futures in the Korean Futures Exchange market and obtained that the decay distribution for the survival probability is exhibited novel stretched-exponential forms. Scalas et al. [14,15] applied the CTRW theory to models of the high-frequency price dynamics. They reported that the survival probability for high-frequency data of 30DJIA stocks shows an non-exponential decay.

The purpose of this paper is to present theoretical and numerical arguments for the yen–dollar exchange rate traded at the Japanese exchange market, and we apply the formalism of the CTRW theory to financial tick data of the yen–dollar exchange rate. In Section 2, we introduce stochastic quantities included the volatility in the CTRW theory. In the final section, the dynamical behavior of returns and volatilities for the yen–dollar exchange rate is mainly calculated numerically at the long-time limit, and we conclude with some results and outlook.

2. Volatilities in the CTRW

In this section, we will mainly focus on the CTRW theory in order to discuss the probability density function. Let $R(t)$ be the return defined by

$$R(t) = \ln\left[\frac{P(t+t_0)}{P(t_0)}\right], \tag{1}$$

where $P(t)$ is the price of the yen–dollar exchange rate at time t, and t_0 is an arbitrary time. The return $Y(t)$ becomes

$$Y(t) = R(t) - \langle R(t)\rangle . \tag{2}$$

We introduce the pausing time density function and the transition probability as follows: when $X_n = t_n - t_{n-1}$ and $Y_n = Y(t_n) - Y(t_{n-1})$, the pausing time density function and the jumping probability density function are, respectively,