Physica A 389 (2010) 4623–4635

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

A higher-order macroscopic model for pedestrian flows

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ARTICLE INFO

Article history:
Received 21 November 2009
Received in revised form 24 March 2010
Available online 23 May 2010

Keywords:
Pedestrian crowd dynamics
Path choice
Linear stability analysis
Traffic instability
Unstructured meshes
Obstruction

ABSTRACT

This paper develops a higher-order macroscopic model of pedestrian crowd dynamics derived from fluid dynamics that consists of two-dimensional Euler equations with relaxation. The desired directional motion of pedestrians is determined by an Eikonal-type equation, which describes a problem that minimizes the instantaneous total walking cost from origin to destination. A linear stability analysis of the model demonstrates its ability to describe traffic instability in crowd flows. The algorithm to solve the macroscopic model is composed of a splitting technique introduced to treat the relaxation terms, a second-order positivity-preserving central-upwind scheme for hyperbolic conservation laws, and a fast-sweeping method for the Eikonal-type equation on unstructured meshes. To test the applicability of the model, we study a challenging pedestrian crowd flow problem of the presence of an obstruction in a two-dimensional continuous walking facility. The numerical results indicate the rationality of the model and the effectiveness of the computational algorithm in predicting the flux or density distribution and the macroscopic behavior of the pedestrian crowd flow. The simulation results are compared with those obtained by the two-dimensional Lighthill–Whitham–Richards pedestrian flow model with various model parameters, which further shows that the macroscopic model is able to correctly describe complex phenomena such as "stop-and-go waves" observed in empirical pedestrian flows.

1. Introduction

The simulation of pedestrian and crowd dynamics has become increasingly important for the security and safety management of pedestrian traffic. Numerical simulation of crowds, both as a continuum and in terms of discrete pedestrians, is an effective tool to investigate and predict the characteristics of crowd behavior and movement [1–23], such as the formation of lanes of uniform walking directions and oscillations at bottlenecks at moderate densities. The idea of treating the flows of large crowds of pedestrians as continuous media with a path choice decision process is a recent development in pedestrian studies [17,18]. With this approach, individual behavior is averaged out and collective behavior, in particular pedestrian movement, can be taken into account, thus allowing macroscopic modeling. The macroscopic models in the literature describe the dynamics of macroscopic variables (e.g., density, velocity, and flow) using a set of partial differential equations [16–23]. Among these models, the two-dimensional (2D) Lighthill–Whitham–Richards (LWR) model [18,21,23] has received much attention. The model describes the conservation of mass by assuming an equilibrium state of the flow–density relationship and a directional motion of an individual pedestrian that minimizes his or her instantaneous travel cost to a destination. Numerical simulations of the LWR model have shown it to be a useful tool for the planning and design of walking facilities. Non-equilibrium phase transitions and various nonlinear dynamic phenomena such as the formation

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doi:10.1016/j.physa.2010.05.003

of traffic jams and stop-and-go waves were observed in empirical pedestrian flows [24,25], and it is thus reasonable that 2D pedestrian dynamic equations can be extended from the one-dimensional (1D) higher-order vehicular continuum models to describe the complex phenomena in real traffic [2,20]. The main difference between crowd and vehicular flows is that the former can walk freely in a 2D continuous domain, whereas the latter move in a 1D space. This means that pedestrians can make a simultaneous path choice, based on a specific traffic situation.

This paper aims to simulate a macroscopic pedestrian dynamic model derived from fluid dynamics that consists of 2D Euler equations with relaxation. The dynamic model describes the conservation of mass and the equilibrium of linear "momentum". The model can be viewed as an extension of the 1D Payne–Whitham (PW) vehicular model [26,27]. Its homogeneous equations remain hyperbolic and isotropic. The desired direction of motion appearing in the relaxation terms of the model needs to be specified. We assume that pedestrians tend to move in a reactive user-optimal manner whereby they choose the path that minimizes the instantaneous total travel cost from origin to destination. The travel cost mainly represents the travel time, and the cost distribution is thus defined as the inverse of the speed. The assumption gives rise to an Eikonal-type equation [21,23] by which the desired direction of motion can be determined. A linear stability analysis of the dynamic model shows that the model can describe traffic instability in crowd flows. For large enough perturbations in a crowd flow such as a traffic accident or bottleneck, the condition of stability is violated and traffic instability occurs, which leads to non-equilibrium phase transitions and the nonlinear dynamical phenomena observed in empirical pedestrian flows [24].

Unstructured meshes are used for the spatial discretization of the model to allow the modeling of a continuous walking facility with a complex geometry. To obtain a more stable numerical algorithm to solve the model, a splitting technique [28] is applied to divide the model into a homogeneous subproblem and a source subproblem for a given time increment. The algorithm comprises two major steps. In the first, a second-order fast-sweeping method (FSM) is applied to solve the Eikonal-type equation, which generates the desired direction of motion. In the second, a cell-centered finite-volume method (FVM) coupled with a second-order positivity-preserving central-upwind scheme is designed to derive the semi-discretized scheme of the balance laws. The resultant scheme is of second-order accuracy, which matches the second-order total variation diminishing (TVD) Runge–Kutta method for time discretization. We use an example of a pedestrian crowd flow problem in the presence of an obstruction within a 2D walking facility to demonstrate the applicability of the model and the effectiveness of the computational algorithm. The numerical results for the formation of high-density regions and the effects of the obstruction on the flow characteristics indicate that the model produces reasonable patterns of pedestrian movement, and that the solution algorithm is effective. These results also help to visualize the evolution of a crowd and the motion trajectories of pedestrians, and to predict the macroscopic characteristics of crowd flows. The simulation results based on various model parameters are compared with those obtained by the 2D LWR model [18,23] for pedestrian flow, which further demonstrates that the macroscopic model is able to correctly describe the complex phenomena in empirical pedestrian flows.

The remainder of this paper is organized as follows. In Section 2, we derive the pedestrian dynamic model from fluid dynamics and its linear stability condition. Section 3 deals with the numerical algorithm used to solve the model. In Section 4, we consider a numerical example to test the rationality of the model and the efficiency of the algorithm. The final section offers some conclusions.

2. Pedestrian dynamic model

In this section, a pedestrian dynamic model is formulated as a set of 2D Euler equations with relaxation. A linear stability analysis is performed on the dynamic model, which shows the model's ability to describe traffic instability in a crowd flow. The model can be viewed as an extension of the 1D Payne–Whitham (PW) vehicular model, and its homogeneous equations remain hyperbolic and isotropic.

2.1. Model description

Consider pedestrians in a crowd moving through a 2D continuous walking facility Ω (in m^2) that contains an obstruction. The wall of Ω and the sections of the entrance and exit are respectively denoted by Γ_w, Γ_i, and Γ_o (in m). Similar to flow conservation in fluid mechanics, the traffic density and flow vector must satisfy the following flow conservation equation:

$$\rho_t + \nabla \cdot \mathbf{F} = 0, \quad \forall (x, y) \in \Omega, t \in T, \tag{1}$$

where $\rho_t = \frac{\partial \rho}{\partial t}$ and $\nabla = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y})$; T (in s) is the modeling period; $\rho(x, y, t)$ (in ped/m^2) denotes the pedestrian density at location (x, y) at time t; and $\mathbf{F} = \rho\mathbf{v}$ is the flow vector (in ped/m/s), where $\mathbf{v} = (u(x, y, t), v(x, y, t))$ (in m/s) are the average speeds of pedestrian motion in the x-direction and the y-direction, respectively.

The differences between the various existing macroscopic traffic models relate to the equations for the average pedestrian velocity. The 2D LWR model assumes an equilibrium state of the speed–density relationship $U_e(\rho)$ that is dependent on the local pedestrian density ρ, and a corresponding directional motion of an individual pedestrian $\boldsymbol{\nu} = (\nu_x(x, y, t), \nu_y(x, y, t))$. To describe complex pedestrian traffic phenomena such as traffic jams and stop-and-go phenomena [24], dynamic equations for the average speeds u, v must be introduced by using a similar approach to that applied to vehicle traffic flow. We refer the reader to [29–31] for discussion of the mentioned phenomena along with their analytical properties in the vehicle traffic flow problem.