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Cross-correlation markers in stochastic dynamics of complex systems

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ABSTRACT

The neuromagnetic activity (magnetoencephalogram, MEG) from healthy human brain and from an epileptic patient against chromatic flickering stimuli has been earlier analyzed on the basis of a memory functions formalism (MFF). Information measures of memory as well as relaxation parameters revealed high individuality and unique features in the neuromagnetic brain responses of each subject. The current paper demonstrates new capabilities of MFF by studying cross-correlations between MEG signals obtained from multiple and distant brain regions. It is shown that the MEG signals of healthy subjects are characterized by well-defined effects of frequency synchronization and at the same time by the domination of low-frequency processes. On the contrary, the MEG of a patient is characterized by a sharp abnormality of frequency synchronization, and also by prevalence of high-frequency quasi-periodic processes. Modification of synchronization effects and dynamics of cross-correlations offer a promising method of detecting pathological abnormalities in brain responses.

1. Introduction. Synchronization and collective effects in time series analysis of complex systems

One of the main factors determining the evolution of complex systems is the presence of collective effects arising from an interacting or redistributing of the certain connections between parts of a composite system. In many cases it is impossible to make an adequate analysis of the functioning of such systems by ignoring the underlying collaborative mechanisms.

There are various approaches used in studying the collective phenomena in complex systems. Somehow or other, all of them are based on the analyzing unique features of the connected systems: certain quantitative and qualitative ratios between the system elements, a dynamic coordination of components under the external influences, specific synchronization phenomena. Some recent results have been derived by studying the effects of frequency and phase synchronization [1–5]. These methods are based on revealing the characteristic frequencies and analyzing the differences in the phases of dynamic variables derived by means of the Fourier transform, Hilbert transform [1,2,4] and wavelet-transform [5]. Within the framework of another methodology the stochastic synchronization is studied by comparing topological structures of attractors, describing the dynamics of two nonlinear coupled oscillators [6]. The "generalized synchronization" relationship [7] also uses the topological method and is the successful original development of the stochastic synchronization approach.

Another approach to study the collective effects in complex systems is the analysis of cross-correlations, i.e. the probabilistic relation between the sequences of random variables. The cross-correlation method is used to describe the collective phenomena in various systems (physical, economical, biological and physiological). The perspective approach in

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this field of research is detrended cross-correlation analysis, which was introduced in Ref. [8] to study the power-law cross-correlations between nonstationary time series of various nature. This method is widely used in financial systems [9–11].

There are several alternative methods to analyze cross-correlations, particularly, random matrix theory [12], approximate entropy [13] and sample entropy [14]. In Ref. [12], the authors used random matrix theory, a method originally developed to study the spectra of complex nuclei, to analyze the mutual dynamics in price changes of the stocks. In Ref. [13], the authors used the method of approximate entropy, a model independent measure of sequential irregularity which is based on Kolmogorov entropy, as an indicator of system stability. In Ref. [14], the authors proposed the sample entropy, a modified and unbiased version of approximate entropy, as a measure of degree of asynchrony in physiological signals.

Although these earlier methods find useful applications in real-life time series, there exists another set of methods in cross-correlation analysis which uses the cross-correlation functions itself [3,15,16]. In paper [3], the authors compared the linear synchronization measures including the cross-correlation functions to nonlinear ones and revealed that for the considered experimental data all measures ranked the synchronization levels of the three examples in the same way. In work [15], the authors develop a theory of neuronal cross-correlation functions for analyzing the neuron interactions in large neural networks including several highly connected sub-populations of neurons. In [16], the authors developed a method based on the model of fractional Brownian motion and Hurst exponent to describe the coupling of the non-stationary signals with long-range correlations.

In this study, we demonstrate the fundamentally new opportunities of studying the collective effects in complex systems. Our method is based on generalization of the memory functions formalism [17,18] in a case of cross-correlations between the spaced elements of the studied system. The important advantage of this method is the description of the cross-correlations in different relaxation scales in the time series of a complex system. Here, we consider neuromagnetic responses (magnetoencephalogram, MEG) of the brain as a suitable example of a time series of a complex system, i.e. the human brain. Earlier analysis of MEG signals [19] were performed on the basis of the memory functions formalism (MFF), and revealed an important autocorrelation difference between MEG signals of healthy subjects and of a patient with photosensitive epilepsy (PSE). Particularly, this difference appears in qualitative alterations of the power spectra of memory functions. Besides, it has been shown that the statistical memory effects play a key role in identification of PSE.

Here we investigate cross-correlations in the MEG responses simultaneously obtained from multiple brain regions. We will show that mechanisms of formation of the PSE are connected, first of all, with abnormality of interrelations between the spaced areas of a cerebral cortex, which result in suppression of its regulator functions at formation of the response to external influences. The paper is structured as follows. Section 2 presents the basic relations of the memory functions formalism for cross-correlations. Section 3 details the experimental details. Section 4 contains our results including the calculation of cross-correlation functions, memory functions and their power spectra. Section 5 offers general conclusions about connections between the PSE pathological alterations and suppression of coordination effects (frequency-phase synchronization).

2. Basic relations of the memory function formalism in the case of cross-correlations

Following [17–20], we consider the stochastic dynamics of the magnetic induction gradient, registered in two different brain regions as the sequences $\{x_j\}$, $\{y_j\}$ of random values X, Y:

$$\begin{aligned} X &= \{x(T), x(T+\tau), x(T+2\tau), \dots, x(T+(N-1)\tau)\},\\ Y &= \{y(T), y(T+\tau), y(T+2\tau), \dots, y(T+(N-1)\tau)\}, \end{aligned} \tag{1}$$

where T is the initial time point, $(N-1)\tau$ is the time period of signal registration, τ is the time interval of signal discretization.

Mean values, fluctuations and dispersions for a set of random values (1) can be written as follows

$$\langle X\rangle = \frac{1}{N}\sum_{j=0}^{N-1} x(T+j\tau), \qquad x_j = x(T+j\tau), \qquad \delta x_j = x_j - \langle X\rangle, \qquad \sigma_x^2 = \frac{1}{N}\sum_{j=0}^{N-1}\delta x_j^2;$$

$$\langle Y\rangle = \frac{1}{N}\sum_{j=0}^{N-1} y(T+j\tau), \qquad y_j = y(T+j\tau), \qquad \delta y_j = y_j - \langle Y\rangle, \qquad \sigma_y^2 = \frac{1}{N}\sum_{j=0}^{N-1}\delta y_j^2.$$

To describe the probabilistic relation between the sequences of random variables X and Y we use the normalized time-dependent cross-correlation function (CCF):

$$c(t) = \frac{1}{(N-m)\sigma_x\sigma_y}\sum_{j=0}^{N-m-1}\delta x(T+j\tau)\delta y(T+(j+m)\tau), \quad t = m\tau,\ 1\le m\le N-1. \tag{2}$$

Function (2) satisfies the conditions of normalization and relaxation of correlations:

$$\lim_{t\to 0} c(t) = 1, \qquad \lim_{t\to\infty} c(t) = 0.$$

It should be noted that the second property is not always satisfied for the real time series of complex systems.