Physica A 389 (2010) 5053–5060

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Height–height correlations for surface growth on percolation networks

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ARTICLE INFO

Article history:
Received 26 October 2009
Received in revised form 5 June 2010
Available online 28 June 2010

Keywords:
Surface growth
Restricted solid-on-solid model
Percolation network
Height–height correlation function
Critical exponents

ABSTRACT

The height–height correlations of the surface growth for equilibrium and nonequilibrium restricted solid-on-solid (RSOS) model were investigated on randomly diluted lattices, i.e., on infinite percolation networks. It was found that the correlation function calculated over the chemical distances reflected the dynamics better than that calculated over the geometrical distances. For the equilibrium growth on a critical percolation network, the correlation function for the evolution time $t \gg 1$ yielded a power-law behavior with the power ζ', associated with the roughness exponent ζ via the relation $\zeta = \zeta' d_f/d_l$, with d_f and d_l being, respectively, the fractal dimension and the chemical dimension of the substrate. For the nonequilibrium growth, on the other hand, the correlation functions did not yield power-law behaviors for the concentration of diluted sites x less than or equal to the critical concentration x_c.

1. Introduction

Surface roughening is associated with a wide variety of systems, such as domain walls of a two-dimensional random bond Ising model [1], randomly stirred fluids [2], ballistic aggregation [3], and directed polymers in a random potential [4,5], and has been extensively studied by using various continuum growth equations as well as discrete atomistic models [6,7]. The most common continuum equation describing the surface growth is the Kardar–Parisi–Zhang (KPZ) equation, given as [8]

$$\frac{\partial h}{\partial t} = \nu \nabla^2 h + \frac{\lambda}{2} (\nabla h)^2 + \eta(\vec{r}, t), \tag{1}$$

where η represents the Gaussian random variable that satisfies

$$\langle \eta(\vec{r}, t) \eta(\vec{r}', t') \rangle = 2\Gamma \delta(\vec{r} - \vec{r}') \delta(t - t'), \tag{2}$$

with Γ describing the local noise variation. Various discrete models such as the Eden growth [9], the ballistic deposition [10], and the restricted solid-on-solid (RSOS) model [11] were known to be described by the KPZ equation.

The coefficient λ in the "continuum" KPZ equation describing the curvature of the local growth velocity is associated with the difference of the deposition rate and the evaporation rate in the discrete lattice models [12]. Therefore, for the symmetric case where the two rates are equal, λ should vanish and the KPZ equation is reduced to the Edwards–Wilkinson (EW) equation [13]. On the other hand, for the asymmetric case where the two rates are not equal, the growth becomes nonequilibrium in nature, and the critical behavior is expected to be the same as for the pure deposition model. The models that are described by the EW equation are considered to belong to the EW universality class, whereas those which are described by the KPZ equation are considered to belong to the KPZ universality class.

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doi:10.1016/j.physa.2010.06.039

The surface width, W, is defined as the standard deviation or the root-mean-square fluctuation of heights:

$$W(t,L) = \left\langle (h(\vec{r},t) - \overline{h(t)})^2 \right\rangle^{1/2}, \tag{3}$$

where $h(\vec{r},t)$ represents the local height variable at site $\vec{r}$ and time t, $\overline{h(t)}$ represents the average height over lattice sites, and $\langle\cdots\rangle$ denotes the average over various samples. Here, d is the substrate dimension and, therefore, the total dimension is $d+1$. It is generally hypothesized that $W(t,L)$ obeys the scaling relation, given as [14]

$$\begin{aligned} W(t,L) = L^{\zeta} f(t/L^z) &\to t^{\beta}, \quad t \ll L^z, \\ &\to L^{\zeta}, \quad t \gg L^z, \end{aligned} \tag{4}$$

where the scaling function $f(x)$ behaves as x^{β} for $x \ll 1$ and becomes constant in the $x \gg 1$ limit. The roughness exponent ζ is the quantity that describes the characteristic of the saturated surface width at a sufficiently late time. The growth exponent β and the dynamic exponent z are associated with each other by the relationship $z\beta = \zeta$. In the equilibrium growth with $\lambda = 0$, the EW equation was solved exactly, yielding $\beta = (2-d)/4$, $\zeta = (2-d)/2$ and $z = 2$ [13]. In the nonequilibrium growth with $\lambda \neq 0$, the KPZ equation was solved only in one dimension, giving $\beta = \frac{1}{3}$ and $\zeta = \frac{1}{2}$ [8]. In 2+1 dimensions, the exponents were obtained by numerical simulations as $\zeta = 0.38$–0.40 and $\beta = 0.24$–0.25 [5,11,15–17]. From the simulation results in one to four dimensions, the exponents were conjectured to be $\beta = 1/(d+1)$ and $\zeta = 2/(d+2)$ in a d-substrate dimension [11,12], but they were not yet confirmed in higher dimensions [17–20].

Besides the growth of surface width, the height–height correlation function defined by the mean-square difference of heights separated by a distance r,

$$G(r,t) = \langle [h(r_0 + r, t) - h(r_0, t)]^2 \rangle, \tag{5}$$

is also an interesting quantity that describes dynamics of the growing surfaces [8,11,21]. For $r \ll \xi$, since the heights within this distance are fully correlated, the correlation function increases as $G(r,t) \sim r^{2\zeta}$. As time increases, the correlation length ξ increases as $\xi \sim t^{1/z}$ and eventually reaches L in the $t \gg L^z$ limit. Within the time $t \ll L^z$ for $r \gg \xi$, it is easy to show that $G(r,t) = 2W^2(t)$ and, therefore, $G(r,t) \to G_{\rm sat}(t) \propto t^{2\beta}$. Therefore the height–height correlation function scales as

$$\begin{aligned} G(r,t) = r^{2\zeta} g(r/t^{1/z}) &\to r^{2\zeta}, \quad r \ll t^{1/z}, \\ &\to t^{2\beta}, \quad r \gg t^{1/z}, \end{aligned} \tag{6}$$

where the scaling function $g(u)$ is constant for $u \ll 1$ and is $u^{-2\zeta}$ for $u \gg 1$.

Recently, surface growth for the equilibrium and nonequilibrium RSOS models was studied on diluted lattices, i.e., on percolation networks, generated for the occupation probability of percolation $p \leq p_c$, and the growth and roughness exponents, β and ζ, were measured from the surface width [22]. It was found that, for the equilibrium growth, the critical exponents were unchanged for the concentration of diluted sites $x\,(=1-p)$ below the critical concentration x_c. For $x = x_c$ $(=1-p_c, p_c$ being the percolation threshold), the growth yielded nontrivial exponents which were different from those on a regular lattice. For the nonequilibrium growth, on the other hand, a considerable amount of diluted sites $x \leq x_c$ appeared to break the power-law behaviors for both the surface width against time and the saturated width against system size.

Motivated by these results, an influence of the diluted sites on the height–height correlation function is investigated in this work. The attention is focused particularly on how the diluted sites influence the power-law behavior of $G(r,t)$ in the two extreme limits, for both the equilibrium and nonequilibrium growth in two substrate dimensions. The diluted sites are considered to be the quenched disorder, and influence of such a quenched disorder has been of a long-standing problem in various problems in statistical mechanics [23,24].

In Section 2, the models and the simulation methods are described. In Sections 3 and 4, results of the equilibrium and nonequilibrium growths for the RSOS model are presented. Summary and concluding remarks are provided in Section 5.

2. Models and methods

In this section, the equilibrium and nonequilibrium RSOS growth models and the method of sampling for the correlation functions are described.

2.1. Restricted solid-on-solid model

The dynamic rule of the RSOS condition is to randomly select a site at $\vec{r}$ and deposit a particle with a probability p_+, or evaporate with a probability p_-, i.e., $h(\vec{r}) \to h(\vec{r}) \pm 1$, provided that the restriction regarding the local height difference

$$|\nabla h| = |h(\vec{r}) - h(\vec{r}')| \leq 1 \tag{7}$$

is satisfied between the selected site at $\vec{r}$ and the nearest-neighbor sites at $\vec{r}'$. In the equilibrium RSOS model, $p_+ = p_- = \frac{1}{2}$ and the strength of nonlinearity vanishes, i.e, $\lambda = 0$. Thus, the model is described by the EW equation. It has been known