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Metal insulator transition in modulated quantum Hall systems

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Abstract

The quantum Hall effect is studied numerically in modulated two-dimensional electron systems in the presence of disorder. Based on the scaling property of the Hall conductivity as well as the localization length, the critical energies where the states are extended are identified. We find that the critical energies, which are distributed to each of the subbands, combine into one when the disorder becomes strong, in the way depending on the symmetry of the disorder and/or the periodic potential.

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1. Introduction

The localization due to the disorder plays a crucial role in the integer quantum Hall effect. In a two-dimensional (2D) electron system in strong magnetic fields, the weak disorder makes almost all the states localized, and the Hall current is carried only by the extended states left at the center of the Landau band. The Hall conductivity is exactly quantized when the Fermi energy lies on the localized region.

When the 2D electron system is subjected to a two-dimensionally modulated potential, the energy spectrum has a recursive gap structure called the Hofstadter butterfly in each of the Landau levels [1]. The system exhibits the quantum Hall effect when the Fermi energy is in each of those gaps, where the intricate gap structure leads to a nontrivial sequence of the quantized Hall conductivity [2]. The nonmonotonic behavior of the Hall conductivity peculiar to this system was experimentally observed in lateral superlattices patterned on GaAs/AlGaAs heterostructures [3,10].

It is an intriguing problem how the Hall conductivity is quantized when the disorder is introduced to the Hofstadter butterfly. The localization problems in the disordered butterfly system have been studied by several authors. A finite-size scaling analysis was performed and the

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critical exponent was estimated at the center of the Landau level [4]. A qualitative discussion on the evolution of the extended states in the Hofstadter butterfly as a function of the disorder for several flux states was given [5,6].

Recently the Hall conductivity σ_{xy} was calculated and the effect of the localization on the quantum Hall effect was studied in the Hofstadter butterfly [7]. It was shown that σ_{xy} becomes independent of the system size at $\sigma_{xy} = \frac{1}{2}$ (in units of $-e^2/h$), and those fixed points can be identified as the critical energies in an infinite system. While the systems in the previous work have an electron–hole symmetry between positive and negative energies, we study here the case without this symmetry to see whether the fixed points are still on $\sigma_{xy} = \frac{1}{2}$, and how the asymmetry affects the evolution of the critical energies as a function of disorder. The electron–hole symmetry occurs when both the periodic and the disorder potentials are symmetric with respect to zero energy. We first consider a system with an asymmetric disorder potential containing only positive scatters, while the periodic potential is left symmetric. Second, we recover the symmetry for the disorder, but make the periodic potential asymmetric as expected in the presence of the electron screening.

2. Formulation

We consider a 2D system in a strong magnetic field with a periodic potential V_{p} and a disorder potential V_{d},

$$H = \frac{1}{2m}(\boldsymbol{p} + e\boldsymbol{A})^2 + V_{\mathrm{p}} + V_{\mathrm{d}}.$$

The band structure is characterized by a parameter $\phi = Ba^2/(h/e)$, a number of magnetic flux quanta penetrating unit cell [1]. We assume that V_{p} has a square form

$$V_{\mathrm{p}} = V\left(\cos\frac{2\pi}{a}x + \cos\frac{2\pi}{a}y\right) + V'\left(\cos\frac{4\pi}{a}x + \cos\frac{4\pi}{a}y\right),$$

where V' represents a double period component breaking the electron–hole symmetry between positive and negative energies. The disorder potential is taken as randomly distributed delta-potentials $\pm v_0$, where the amounts of the positive and negative scatterers are given by N_+ and N_-, respectively. The energy scale for the disorder is given by $\Gamma = 4n_i v_0^2/(2\pi l^2)$, where l is the magnetic length and n_i is the number of the scatterers in a unit area. We consider only the lowest Landau level, assuming that the magnetic field is strong enough and the mixing of the Landau levels is neglected.

We calculate the Hall conductivity using the Kubo formula for zero temperature,

$$\sigma_{xy} = \frac{\hbar e^2}{iL^2} \sum_{\varepsilon_\alpha < E_{\mathrm{F}}} \sum_{\beta \neq \alpha} \times \frac{\langle\alpha|v_x|\beta\rangle\langle\beta|v_y|\alpha\rangle - \langle\alpha|v_y|\beta\rangle\langle\beta|v_x|\alpha\rangle}{(\varepsilon_\alpha - \varepsilon_\beta)^2},$$

where ε_α is the energy of the eigenstate $|\alpha\rangle$, E_{F} the Fermi energy, v_i the velocity operator, and L is the system size.

3. Results

We first consider a system with symmetric potential $V' = 0$ and disorder potential containing only positive scatterers, where the latter breaks the electron–hole symmetry. We assume a magnetic flux $\phi = \frac{3}{2}$, where the lowest Landau level splits into three subbands in the absence of disorder and σ_{xy} for each of them becomes $(1, -1, 1)$ (in units of $-e^2/h$) [2].

Fig. 1 shows numerical results calculated for the disordered systems with $\Gamma = 0.5V$. We show in the panel (a) σ_{xy} for several system sizes with the density of states, (b) the difference in σ_{xy} measured from the smallest sample, and (c) the inverse localization length $1/L_{\mathrm{loc}}$ estimated by the Thouless number method [8], where every quantity is averaged over a number of different samples. We can immediately see that each subband has fixed points in the Hall conductivity at $\sigma_{xy} = \frac{1}{2}$, and they agree with the energies where localization length diverges within the statistics error. σ_{xy} off the fixed points always approaches 1 in the area $\sigma_{xy} > \frac{1}{2}$, and 0 in $\sigma_{xy} < \frac{1}{2}$, leading to the Hall plateau in an