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Deterministic weighted scale-free small-world networks

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ABSTRACT

We propose a deterministic weighted scale-free small-world model for considering pseudofractal web with the co-evolution of topology and weight. Considering the fluctuations in traffic flow constitute a main reason for congestion of packet delivery and poor performance of communication networks, we suggest a recursive algorithm to generate the network, which restricts the traffic fluctuations on it effectively during the evolutionary process. We provide a relatively complete view of topological structure and weight dynamics characteristics of the networks such as weight and strength distribution, degree correlations, average clustering coefficient and degree-cluster correlations as well as the diameter.

1. Introduction

To understand the general principles in architectures of networks, many deterministic models are introduced into complex networks [1–21]. These models are useful tools for investigating analytically not only topological features of networks in detail [1–12], but also dynamical problems on the networks [13–16]. Before presenting our own findings, it is worth reviewing some of this preceding work to understand its achievements and shortcomings. Deterministic scale-free networks were firstly proposed by Barabási et al. in Ref. [1] and intensively studied in Ref. [2] to generate a scale-free topology. However, to some extent, the small exponent γ of the degree distribution for the model did not satisfy the real statistic results well. Instead, Dorogovtsev et al. introduced another elegant model, called pseudofractal scale-free web (PSW) [3] which is extended by Comellas et al. consequently [4]. Based on a similar idea of PSW, Jung et al. presented a class of recursive trees [5], which have the small-world behavior built in. Additionally, in order to discuss modularity, Ravasz et al. proposed a hierarchical network model [6,7], the exact scaling properties and extensive study of which were reported in Refs. [8,9], respectively. Recently, motivated by the problem of Apollonian space-filling packing, Apollonian networks [10] with a typical loop structure were introduced and intensively investigated [17–21]. These pioneering works are all invaluable tools for the topology of network studies.

In the last few years, it is found that many real networks are inhomogeneous, consisting of distinct nodes and links. For instance, the scientist collaboration network, where scientists are identified with nodes, and an edge exists between two scientists if they have coauthored at least one paper [22], and the Internet at the AS level, where the link weights represent the bandwidth of a cable and node weight the load of a router [23], among other areas. Recently, weight dynamics ideas have been applied with success to topics as diverse, such as random walks [24], condensation [25], synchronization [26], traffic congestion [27], epidemic spreading [28,29], information filtering [30], to name but a few. The findings above might

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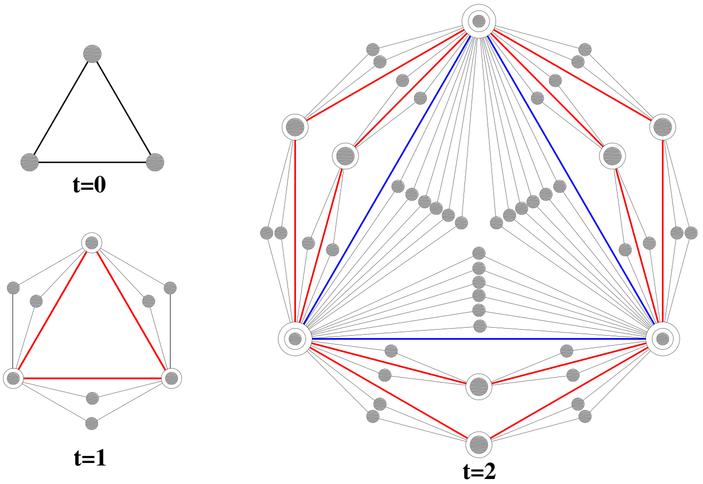

Fig. 1. Illustration of the deterministically growing network for the particular case of $m = 2$ and $\delta = 1$, showing the first three steps of growing process. The gray links in the figure denote the links with weight 1, the red links with weight 3, and the blue links with weight 9. The number of rings around a gray node denotes its age. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

provide insight for understanding the correlations among weighted quantities and the underlying topological structure and dynamics behaviors of the weighted networks.

Most previous weighted random models [31–35] with topology and weight co-evolution, however, possess very loose clustering structures when the size of the networks is large. At the same time, previous deterministic models, are mainly unweighted [1–21], which ignore the heterogeneity of edges in real networks. What is more, the models [3,4] on PSW networks fail to provide the reason for adopting the recursive way to build up the networks. Consequently, in this paper, we introduce a model bringing weight evolution into the growth of pseudofractal scale-free web (PSW) [3] that aims to circumvent these incongruities properly. As we will show, in the case of the recursive construction, the traffic and its fluctuation decrease exponentially with time either on edges or at nodes. Hence, we believe the construction method may shed some light on network design to improve the control and speed of the whole network [36]. At the same time, our comprehensive and rigorous solutions may help people understand better the interplay between network topology and weight dynamics.

2. The model

The construction of the model is controlled by two parameters m and δ, evolving in a recursive way. We denote the network after t steps by $G(t)$, $t \geq 0$(see Fig. 1). Then the network at step t is constructed as follows. For $t = 0$, $R(0)$ is a triangle consisting of three links with unit weight. For $t \geq 1$, $G(t)$ is obtained from $G(t-1)$. We add mw (m is positive integer) new nodes for each of the links with weight w, and we connect each new node to both ends of this link by new links of unit weight; moreover, we increase the weight of these links by $m\delta w$ (δ is positive integer).

Before introducing our model further, we explain why adopting such a recursive way and why the generated network is increasingly efficient for transmitting information with network order. In this model, the recursive construction is motivated by the practical need to improve the transport capacity of real networks. As is known to us, both the physical networks and the numbers of users are growing continuously. The performance of the networks for larger system sizes and heavier loads are critical issues to be addressed in order to guarantee networks' functioning in the near future. For example, if the traffic fluctuates dramatically, a highway is more likely to be congested frequently when the peak value of traffic exceeding its capacity. There is thus a need to build up more branches to distribute the heavy traffic. However, "how many branches must we have?" and "Where shall we put them?" are open questions yet.

Recently, the authors of Ref. [37] claimed the fluctuations in traffic flow constitute the main factor affecting the performance of networks. They derived the dependence of fluctuations with the mean traffic on unweighted networks analytically. Consequently, their recipes were adopted extensively to the weighted networks by the authors of Ref. [38]. As shown in Ref. [38], for correlated networks (assortative or disassortative mixing [39,40]), the average traffic through a link L_{ij} during a time window can be represented as

$$\langle f_{ij} \rangle = \frac{2\, w_{ij}}{\sum\limits_{i=1}^{N} s_i} RM, \tag{1}$$

and the standard deviation can be expressed as

$$\sigma_{ij}^2 = \langle f_{ij} \rangle \left(1 + \langle f_{ij} \rangle \frac{\Delta^2 + \Delta}{3R^2} \right), \tag{2}$$