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Obtaining critical point and shift exponent for the anisotropic two-layer Ising and Potts models: Cellular automata approach

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Abstract

Using probabilistic cellular automata with the Glauber algorithm, we have precisely calculated the critical points for the anisotropic two-layer Ising and Potts models ($K_x \neq K_y \neq K_z$) of the square lattice, where K_x and K_y are the nearest-neighbor interactions within each layer in the x and y directions, respectively and K_z is the inter-layer coupling. A general equation is obtained as a function of the inter- and intra-layer interactions (ξ, σ) for both the two-layer Ising and Potts models, separately, where $\xi = K_z/K_x$ and $\sigma = K_y/K_x$. Furthermore, the shift exponent for the two-layer Ising and Potts models is calculated. It was demonstrated that in the case of $\sigma = 1$ for the two-layer Ising model, the value of $\phi = 1.756$ (± 0.0078) supports the scaling theories' prediction that $\phi = \gamma$. However, for the unequal intra-layer couplings for the two-layer Ising model and also in the case of both equal and unequal intra-layer interactions for the two-layer Potts model, our results are different from those obtained from the scaling theories. Finally, an equation is obtained for the shift exponent as a function of intra-layer couplings (σ) for the two-layer Ising and Potts models.

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Keywords: Ising model; Potts model; Cellular automata; Critical point; Shift exponent

1. Introduction

The Ising spin model which is an idealization of magnetic materials has been used to describe a tremendous spectrum of physical and non-physical problems [1–3]. For instance, some excellent manuscripts on the Ising ferromagnet description or the magnetic properties of diluted thin films in the Ising approximation have been published by Balcerzak et al. [4,5]. The general extension of the Ising model is called the Potts model. Interested readers can refer to an excellent review of the Potts model by Wu [6]. Since the exact solution of the Ising model exists only for the one- and two-dimensional models [7,8], the simulation and numerical methods may be used to obtain the critical data for other models. One of the numerical methods is using the transfer matrix and decreasing the matrix size [9]. Ghaemi et al. [10] have used the transfer matrix method to construct the critical curve for a symmetric two-layer

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Ising model. In another work [11], they have used this method to get the critical temperature for the anisotropic two-layer Ising model. Such calculations are limited to lattices with a width of five cells in each layer and the critical point is obtained by the extrapolation approach. There are other numerical methods for solving the Ising model [12]. However, the numerical methods mentioned above are time consuming and advanced mathematics is required when they may be used for extended models like the anisotropic two-layer Potts model.

In most cases, simulation methods are simple and fast. They are also less restricted to the lattice sizes. There are different simulation methods which have been used to describe Ising and Potts models. Monte Carlo is one of the simulation methods which has been widely used for studying the Ising model [13]. In addition, the multicanonical Monte Carlo studies on Ising and Potts models are highly used in recent years [14–16]. The Cellular Automata (CA) are one of methods that could be used to describe the Ising model. The CA are discrete dynamic systems with simple evolution rules that have been proposed as an efficient alternative for the simulation of some physical systems [17]. There are some different approaches which are based on the CA method. The Q2R automaton is an approach which is used for the microcanonical Ising model. It is deterministic, reversible and nonergodic and also a very fast method. Many works have been performed based on this model [18–24]. Although the Q2R model is deterministic and hence is fast, it was demonstrated that the probabilistic model of the CA like Metropolis algorithm [25] is more realistic for the description of the Ising model even though the random number generation makes it slower. The main difference between the Cellular Automata (CA) method and the Monte Carlo method is the updating of a system in each step. In the Monte Carlo method, only one site or a cluster which is randomly chosen is updated in each step. However, in the CA, all sites are updated in each time step without a random selection. In a recent work, Asgari et al. [26] showed that probabilistic CA based on the Glauber algorithm [27] is a fast and reliable simulation method for obtaining the critical point of the two-layer Ising and Potts model in the isotropic case ($K_x = K_y = K_z$), where K_x and K_y are the nearest-neighbor interactions within each layer in the x and y directions, respectively and K_z is the inter-layer coupling. They have also shown that this approach is useful for the case of different inter-layer coupling of the two-layer Potts model ($K_x = K_y \neq K_z$) and constructed a critical curve for this model [28]. Although most of the works that have been done until now are for qualitative descriptions or for introducing fast methods for solving various Ising models, we have shown in our previous works that the probabilistic CA increases the calculations precision [26,28].

In the present work, we have extended our approach to calculate the critical point of an anisotropic two-layer Ising and Potts models ($K_x \neq K_y \neq K_z$) for a square lattice. Thereafter, the obtained results have been fitted in terms of ξ and σ, where $\xi = K_z/K_x$ and $\sigma = K_y/K_x$. The obtained equations are applicable to gain the critical point for these models in terms of the nearest-neighbor interactions within each layer in the x and y directions and also the inter-layer coupling. It is notable that there is no general equation for such cases. Then we have obtained the shift exponent for the two-layer Ising and Potts models for both the cases of equal intra-layer interactions ($K_x = K_y$) and non-equal intra-layer interactions ($K_x \neq K_y$).

2. Two-layer Ising model

The Ising model is a crude attempt to simulate the structure of a physical ferromagnetic substance. The two-layer Ising model, as a simple generalization of the two-dimensional Ising model and also as an intermediate model between the two-dimensional and three-dimensional Ising models, has long been studied [29]. Several approximation methods have been applied to this model [11,30]. Here we have used the CA to obtain the critical point for this model.

A two-layer square lattice with the periodic boundary condition is considered, each layer with p rows and r columns. Each layer has then $r \times p$ sites and the number of the sites in the lattice is $2 \times r \times p = N$. Thereafter the nearest-neighbor interactions are included as well, so the number of neighbors for each site is 5. For any site a spin variable $\sigma^{1(2)}(i, j) = \pm 1$ is defined so that $i = 1, \ldots, r$ and $j = 1, \ldots, p$. So, the average magnetization of the lattice for this model can be defined as [31]

$$\langle M \rangle = \left\langle \sum_{i=1}^{r,*} \sum_{j=1}^{p,*} \sum_{n=1}^{2} \sigma^n(i, j) \right\rangle. \tag{1}$$

We have considered the anisotropic case for a two-layer Ising model i.e. $K_x \neq K_y \neq K_z \geq 0$. We have used a two-layer square lattice with 500×500 sites in each layer with the periodic boundary condition. The Glauber method is used with checkerboard approach to update the sites. Namely, each layer is like a checkered surface and at first,