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Towards effective payoffs in the prisoner's dilemma game on scale-free networks

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Abstract

We study the transition towards effective payoffs in the prisoner's dilemma game on scale-free networks by introducing a normalization parameter guiding the system from accumulated payoffs to payoffs normalized with the connectivity of each agent. We show that during this transition the heterogeneity-based ability of scale-free networks to facilitate cooperative behavior deteriorates continuously, eventually collapsing with the results obtained on regular graphs. The strategy donations and adaptation probabilities of agents with different connectivities are studied. Results reveal that strategies generally spread from agents with larger towards agents with smaller degree. However, this strategy adoption flow reverses sharply in the fully normalized payoff limit. Surprisingly, cooperators occupy the hubs even if the averaged cooperation level due to partly normalized payoffs is moderate.

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1. Introduction

The welfare of a society often demands unselfish behavior of their members. Although working for the common good via cooperation seems a reasonable demand in return for a stable and flourishing society, it is often fundamentally at odds with the Darwinian principles of evolution suggesting that individuals should act exclusively so as to enhance their own prosperity. While understanding the evolution of cooperation among selfish individuals is a puzzle faced by scientists across many different fields of research [1], the mathematical framework of choice for addressing the challenge is often the same, namely the evolutionary theory of games [2–5]. Particularly the prisoner's dilemma game, consisting of cooperators and defectors as the two possible strategies, is considered as a paradigm for studying the emergence of cooperation between unrelated and selfish individuals [6]. The game promises a defecting individual the highest income if facing a cooperator. On the other hand, the cumulative income of two cooperators is higher than

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that of a cooperator–defector pair, and still higher than that of two defectors. According to the fundamental principles of Darwinian selection individuals should decide to defect, which ultimately results in social poverty. This unadorned scenario is indeed described by the classical well-mixed prisoner's dilemma game [3], where the cooperators always die out. However, since the cooperative behavior is an underpinning for a successful society, and is thus arguably present in everyday life [7,8], the gap between the outlined theory and practice is obvious, indeed requiring refinement of the former.

An important milestone in bridging the gap between the outcomes of the well-mixed prisoner's dilemma game and reality was the introduction of spatial structure and nearest-neighbor interactions pioneered by Nowak and May [9], which enabled the cooperators to form cluster on the spatial grid and so protect themselves against being exploited by defectors. The conditions of some biological experiments confirmed this conjecture [10]. However, there also exist compelling evidences that spatial structure may not necessarily favor cooperation [11]. Albeit a statement issued for the snowdrift game, in view of difficulties associated with the payoff rankings in experimental and field work [12,13], it certainly carries an important message and dictates that the search for additional cooperation-facilitating mechanisms is necessary.

Indeed, research performed within recent years has made it clear that, besides spatial extensions and mechanisms such as kin selection, reciprocity, strategic complexity [14], or asymmetry of learning and teaching activities [15], the specific topology of agents defining the interactions among them plays a key role in determining the outcome of the evolutionary process [16–26]. Perhaps most prominently, scale-free (SF) networks [27], previously being identified as omnipresent in man made and natural systems [28], have been established as extremely potent promoters of cooperation in both the prisoner's dilemma as well as the snowdrift game [17,19]. In contrast, some works have shown that the ability of SF networks to promote cooperation disappears if we apply averaged instead of the more widely used accumulated payoffs [29–32]. Moreover, the interplay between the evolution of cooperation as well as that of the network defining the interactions among agents has also received considerable attention [33–38], showing specifically that the evolution of the interaction network might have a beneficial effect on cooperation, and finally, a widely applicable and simple rule for the evolution of cooperation on graphs and social networks has been proposed [39]. For a comprehensive review of the field of research see Ref. [40].

In this paper, we wish to elaborate on the prominent role of scale-free networks and their ability to promote cooperation in the prisoner's dilemma game. More precisely, we study the transition towards effective payoffs in the prisoner's dilemma game on SF networks, whereby the latter is realized via the normalization of the payoffs of each agent with its connectivity. In particular, we introduce a normalization parameter that guides the system continuously from non-normalized, *i.e.* absolute, to normalized, *i.e.* effective, payoffs. A similar but technically different interpolation formula was recently proposed also by Tomassini et al. [30]. However, our goal is not just to demonstrate the deteriorated cooperator successfulness due to payoff normalization, but also to investigate the activity patterns of players that characterize the corresponding stationary states. For this purpose, we introduce strategy donation and adaptation probabilities of agents with different connectivities. These quantities enable us to rigorously identify the direction of the strategy flow. In other words, they serve to explore typical sources and targets characterizing the microscopic strategy adaptation process in the stationary state of the game. We find that cooperator densities of different classes of agents show a relevant dependence on their degree span, thus segregating the agents in terms of the underlying network topology. Moreover, we also study the global intensity of the adaptation process and reveal a resonant-type behavior depending on the payoff normalization parameter.

The paper is structured as follows. Section 2 is devoted to the description of the prisoner's dilemma game and the summarization of the main mechanism leading to an enhanced cooperation on SF networks. Also, it features the introduction of the normalization parameter that is being used to interpolate between the absolute and normalized payoffs. In Section 3 we present the results and discuss biological implications of our findings, whereas in the last Section we summarize our findings and conclude the paper.

2. The game

We consider an evolutionary two-strategy prisoner's dilemma game with agents located on vertices of a scale-free network generated via the celebrated mechanism of growth and preferential attachment [27], yielding a scale-free distribution of their connectivity k. Initially, each vertex x is designated as a cooperator or defector with equal probability. Next, among all k_x neighbors of a randomly chosen vertex x one neighbor y is also chosen at random,