Physica A 389 (2010) 1729–1738

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Freezing transition in a four-directional traffic model for facing and crossing pedestrian flow

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ARTICLE INFO

Article history:
Received 1 September 2009
Received in revised form 12 November 2009
Available online 5 January 2010

Keywords:
Pedestrian flow
Traffic flow
Mean-field approximation
Freezing transition
Instability

ABSTRACT

We study the traffic behavior in the facing and crossing traffic of pedestrians numerically and analytically. There are four kinds of walkers, those moving to east, to west, to north, and to south. We present the mean-field approximation (MFA) model for the four-directional traffic. The model is described in terms of four nonlinear difference equations. The excluded-volume effect and directionality are taken into account. The fundamental diagrams (current–density diagrams) are derived. When pedestrian density is higher than a critical value, the dynamical phase transition occurs from the free flow to the frozen (stopping) state. The critical density is derived by using the linear stability analysis. The velocity and current (flow) at the steady state are derived analytically. The analytical result is consistent with that obtained by the numerical simulation.

1. Introduction

Recently, pedestrian and vehicular traffics have attracted considerable attention [1–5]. Many observed dynamical phenomena in pedestrian and traffic flows have been successfully reproduced with physical methods. The pedestrian flow dynamics is closely connected with the driven many-particle system [6–16]. The typical pedestrian flows have been simulated by the use of a few models: the lattice–gas model of biased random walkers [12–17], the molecular dynamic model of active walkers [6,10,18], and the cellular automaton model [7,8]. Helbing et al. have found that the "freezing by heating" occurs in the pedestrian counter flow by the use of the molecular dynamic model of active walkers [18]. By using the lattice–gas model of biased random walkers, Muramatsu et al. have found independently that the freezing transition occurs from the free flow to the frozen (stopping) state when the pedestrian density is higher than the critical value [17]. The jamming and freezing transitions in the pedestrian counter flow have been studied by some researchers [19–22].

In the freezing transition, pedestrian flow changes from the free traffic to the frozen state in which all pedestrians cannot move by being prevented from going ahead of each other. In the jamming transition, pedestrian flow changes from the free traffic to the jammed traffic in which pedestrians are distributed heterogeneously and move slowly. Thus, the freezing transition is definitely different from the jamming transition.

Similar freezing transition occurs in the two-dimensional cellular automaton traffic model proposed by Biham, Middleton, and Levine (BML model) [23]. There are two kinds of mobile objects in the BML model and the mobile objects moving to east cross with those to north. The BML model is two-directional traffic. The BML model has been extended to various pedestrian traffics and have been studied extensively [24–27].

The theoretical analysis for the freezing transition has been little known. The pedestrian flow has been investigated by the numerical simulation of the self-driven many-particle models. Very recently, the mean-field approximation (MFA) model was proposed. The fundamental diagram and freezing transition point of the facing pedestrian traffic were derived analytically [28].

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doi:10.1016/j.physa.2009.12.038

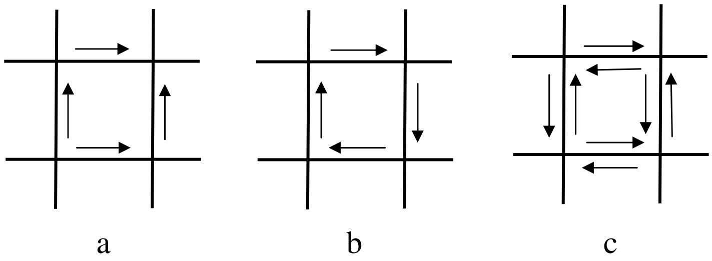

Fig. 1. (a) Schematic illustration of the two-directional traffic model (BML model). (b) Schematic illustration of the four-directional traffic model (extended BML model) without facing. (c) Schematic illustration of the four-directional traffic with facing and crossing. The arrows indicate the moving direction of walkers.

When pedestrians go into the crowd, they move ahead face to face with others and cross with others. Thus, pedestrian flow is at least four-directional traffic with facing and crossing. The BML model is two-directional traffic. The BML model has been extended to four-directional traffic without facing [27]. In the extended model, mobile objects do not move face to face with others on a street (avenue) but go ahead in one direction on each street (avenue) because east (north) and west (south) directional streets (avenues) are positioned alternatively where mobile objects move to east (north) and to west (south) on the east (north) and west (south) directional streets (avenues). In the extended BML model, the facing effect is not taken into account. It is necessary and important to take into account the facing effect in the pedestrian traffic.

In this paper, we present the mean-field approximation (MFA) model of four-directional traffic for the facing and crossing pedestrian flow. We study the dynamical phase transition in the MFA model numerically and analytically. We derive the numerical and analytical solutions for the four-directional pedestrian flow. We present the fundamental diagram (current–density diagram) numerically and analytically. We apply the linear stability method to the MFA model. We derive the freezing transition point analytically. We compare the analytical result with the numerical result.

2. Mean-field approximation model

A pedestrian moves toward his preferential direction with side movement on a street. However, on a coarse-grained scale, the side movement cancels out and pedestrian moves toward his preferential direction on an average. In the mean-field approximation model, the movement for each type of pedestrian is approximated by one-direction motion.

We consider the four-directional traffic of facing and crossing pedestrians on the square lattice. There exist four kinds of walkers: the first is the walkers moving to east, the second the walkers moving to west, the third the walkers moving to north, and the fourth the walkers moving to south. The walkers do not turn. Walkers moving to east go ahead face to face with walkers to west on a street. Walkers moving to north go ahead face to face with walkers moving to south on an avenue. Each walker interacts highly with the other walkers in the front. Fig. 1(c) shows the schematic illustration of four-directional traffic with facing and crossing. The arrows indicate the moving direction of walkers.

Fig. 1(a) shows the two-directional traffic of BML model [23] where there are two kinds of walkers: the first is the walkers moving to east and the second the walkers to north. Walkers do not go ahead face to face but walkers to east cross with walkers to north. Fig. 1(b) shows the four-directional traffic of the extended BML model [27] where the arrows indicate the moving direction on street and avenue. There are four kinds of walkers. However, walkers do not go ahead face to face. The extended BML model cannot be applied to the facing pedestrian traffic. Thus, our model in Fig. 1(c) is definitely different from that in Fig. 1(b). In pedestrian traffic, the facing effect is important.

We consider the mean-field approximation for the four-directional facing pedestrian traffic. We define the probability that a walker to east (to west, to north, and to south) exists on site (i, j) at time t as $p_E(i, j, t)$ ($p_W(i, j, t)$, $p_N(i, j, t)$, and $p_S(i, j, t)$). We apply the conservation law of probability $p_E(i, j, t)$ ($p_W(i, j, t)$, $p_N(i, j, t)$, and $p_S(i, j, t)$) to the four-directional traffic. The probabilities $p_E(i, j, t + \Delta t)$, $p_W(i, j, t + \Delta t)$, $p_N(i, j, t + \Delta t)$, and $p_S(i, j, t + \Delta t)$ of a walker to east, a walker to west, a walker to north, and a walker to south existing on site (i, j) at time $t + \Delta t$ are described, respectively, by the following:

$$p_E(i, j, t + \Delta t) = p_E(i, j, t) + \left[p_E(i - 1, j, t)P_{t,E}(i - 1 \to i, j, t) - p_E(i, j, t)P_{t,E}(i \to i + 1, j, t)\right] \Delta t, \quad (1)$$

$$p_W(i, j, t + \Delta t) = p_W(i, j, t) + \left[p_W(i + 1, j, t)P_{t,W}(i + 1 \to i, j, t) - p_W(i, j, t)P_{t,W}(i \to i - 1, j, t)\right] \Delta t, \quad (2)$$

$$p_N(i, j, t + \Delta t) = p_N(i, j, t) + \left[p_N(i, j - 1, t)P_{t,N}(i, j - 1 \to j, t) - p_N(i, j, t)P_{t,N}(i, j \to j + 1, t)\right] \Delta t, \quad (3)$$

$$p_S(i, j, t + \Delta t) = p_S(i, j, t) + \left[p_S(i, j + 1, t)P_{t,S}(i, j + 1 \to j, t) - p_S(i, j, t)P_{t,S}(i, j \to j - 1, t)\right] \Delta t, \quad (4)$$

where $P_{t,E}(i \to i + 1, j, t)$ is the hopping probability of walker to east from site (i, j) to site $(i + 1, j)$ at time t and $P_{t,W}(i \to i - 1, j, t)$ is the hopping probability of walker to west from site (i, j) to site $(i - 1, j)$ at time t. The second term on the right hand in Eq. (1) represents the inflow of a walker to east from site $(i - 1, j)$ to site (i, j) between t and $t + \Delta t$. The third term represents the outflow of a walker to east from site (i, j) to site $(i + 1, j)$ between t and $t + \Delta t$.