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Equilibrium configurations of uncharged conducting liquid jets in a transverse electric field

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Abstract

Solutions for the problem on the equilibrium configurations of uncharged conducting liquid jets in a transverse electric field are obtained. These solutions correspond to finite-amplitude non-axisymmetric azimuthal deformations of the surface of a round jet: the jet is stretched along the field in its cross-section. The range of electric fields is determined for which solutions of the problem exist. If the electric field strength is over some critical value, the electrostatic equations have no solution, and the jet splits. The obtained solutions are qualitatively examined for stability under small azimuthal perturbations.
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1. Introduction

In the absence of an external electric field, the only equilibrium jet configuration is a round cylinder. Let us consider a jet of conducting fluid in an electric field normal to the jet axis. The force with which the applied electric field acts on the induced surface charge produces a non-axisymmetric azimuthal deformation of the jet surface, stretching it along the field. This is analogous to the behavior of the uncharged drop of a conducting liquid occurring in an electric field (see, e.g. [1,2]). Compensation of the electrostatic forces by the surface tension will lead to a new equilibrium configuration of the jet surface. If the electric field strength is large enough, the electrostatic forces cannot be balanced by capillary forces, and the jet will split into two separate jets. The dynamics of the splitting process and its possible applications for producing small polymeric fibers were recently considered by Paruchuri and Brenner [3]. In particular, they have estimated the splitting time t_s and compared it with the charge relaxation time $t_r > t_s$. In our opinion, the opposite condition $t_s \gg t_r$ should be used; we assume that the liquid in the fluid jet is a perfect electrical conductor (see, e.g. [4]).

It is well known that capillary effects (in the absence of an electric field) lead to the development of the so-called Rayleigh instability of a round cylindrical jet, which is manifested by the growth of longitudinal perturbations whose characteristic wavelength is greater than the length of the jet circumference [5]. A similar

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analysis of the stability of a jet propagating in an electric field is difficult because the unperturbed solution for the jet shape is unknown. Let us compare this situation with the problem concerning geometry of a charged jet. The unperturbed solution for its equilibrium configuration is always known regardless of the linear charge density of the conductor. It is, of course, a cylinder with a circular cross-section. This allows one to analyze the linear stability of the jet [4,6,7] and also to consider the evolution of nonlinear waves propagating on its surface [8–10]. In our case, a round cylinder is no longer the stationary solution. Therefore, the need arises to investigate possible equilibrium configurations of jets in a transverse electric field. Our aim will be (i) to find the equilibrium configurations of strongly deformed jets, (ii) to examine the corresponding solutions for stability, and (iii) to determine the range of electric field strengths where solutions of the problem exist.

Recently, an exact particular solution was found for the special case where the difference of pressures inside and outside the jet is zero [11] (a wide class of exact solutions for the equilibrium configurations of charged jets was obtained in Ref. [12,13]). This solution corresponds to a significant degree of deformation of the jet, such that the aspect ratio of the jet cross-section (A) is $\frac{23}{4}$. It should be noted that the same solution was obtained by McLeod [14] and subsequently studied in detail by Vanden-Broeck and Keller [15], Shankar [16], and Tanveer [17] for a mathematically similar problem of finding the shape of a two-dimensional gas bubble moving in an ideal liquid. As demonstrated in Ref. [11], this solution for the jet shape is unstable with respect to small azimuthal deformations of the surface, and therefore it is of no physical interest (stable configurations of the jet surface correspond to much smaller values of the aspect ratio). Nevertheless, the particular exact solution appears to be useful in seeking approximate solutions of the problem. This is due to the requirements that we impose on the considered approximations of the surface shape. The solutions must be exact for two cases: (i) in the absence of an external field (a round jet, $A = 1$) and (ii) if the difference of pressures inside and outside the jet is zero (the particular solution in Ref. [11], $A = \frac{23}{4}$). This condition enables us to construct an "almost" exact interpolated solution for the jet configurations with $1 < A < \frac{23}{4}$. In the most common situation of a known unique exact solution, the accuracy of the approximation is much lower. Consequently, in the framework of the perturbation theory, if the amplitude of deformation of the exact solution, taken as the perturbation parameter, is small, one should take into account the higher-order terms of the expansions. This would inevitably lead to complicated expressions describing the surface geometry. The simplicity of our interpolated solutions allows their investigation by analytic methods.

2. Initial equations

We assume that the fluid is at rest in the frame of reference moving with the jet and the jet cross-section remains constant along the direction of the jet motion (i.e., we consider only non-axisymmetric azimuthal deformations of the jet surface under the action of electrostatic forces). Let us write down the equations of electrostatics that describe the stationary profile of the surface of an uncharged conducting liquid jet in a transverse electric field of strength E. The distribution of the electric-field potential φ in the plane of the jet cross-section $\{x, y\}$ (in the region outside the jet) is described by the two-dimensional Laplace equation:

$$\varphi_{xx} + \varphi_{yy} = 0.$$

This equation is to be solved subject to the condition that the conductor surface is equipotential, $\varphi = 0$, and to the condition that the electric field at infinity is uniform:

$$\varphi \to -Ey, \quad x^2 + y^2 \to \infty. \tag{1}$$

The equilibrium shape of the surface of a conducting fluid is determined by the balance for the forces acting on the surface

$$(8\pi)^{-1}(\nabla\varphi)^2_{\varphi=0} + TC + P = 0, \tag{2}$$

where the first term describes the electrostatic pressure at the liquid boundary (we use the cgs electrostatic system of units) and the second term describes the surface pressure (T being the surface tension coefficient and C the local curvature of the surface). For a surface given by the parametric expressions

$$y = Y(\tau), \quad x = X(\tau),$$

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