Available online at www.sciencedirect.com
ScienceDirect
Physica A 385 (2007) 161–169
PHYSICA A
www.elsevier.com/locate/physa

Distribution function of fusion reaction products and entropy evolution

D. Mostacci*, V. Molinari, F. Pizzio

Laboratorio di Ingegneria Nucleare di Montecuccolino, Alma Mater Studiorum, Università di Bologna, via dei Colli 16, I-40136 Bologna, Italy

Received 30 April 2007; received in revised form 13 June 2007
Available online 17 June 2007

Abstract

One of the outcomes of nuclear reactions is that reaction products have at birth distribution functions far from Maxwellian. What role do those distribution functions play in the evolution of the entropy of the system? The purpose of this work is to show the effect of the distribution functions of reactant and reaction products on the entropy of a system undergoing DD nuclear fusion reactions. This analysis is conducted with the help of the *H*-theorem, in the framework of kinetic theory. It will be found that at the onset of this reaction, generalized system entropy decreases markedly.

PACS: 05.60.Cd; 05.70.Ln; 05.20.Dd

Keywords: Fusion reactions; Generalized entropy; Kinetic theory; *H*-theorem

1. Introduction

Kinetic theory supplies a bridge between mechanics and thermodynamics [1,2], through which thermodynamic state functions can often be extended to non-equilibrium conditions. The cornerstone of this bridge is Boltzmann's *H*-theorem [2,3], relating the change in entropy per unit volume S_V to the distribution function through the expression

$$\Delta S_V(\mathbf{r}, t) = -K_B \Delta H(\mathbf{r}, t), \tag{1}$$

where K_B is Boltzmann's constant and Boltzmann's *H*-function is given by [2]

$$H(\mathbf{r}, t) = \int_{\Re^3} f(\mathbf{r}, \mathbf{v}, t) \ln f(\mathbf{r}, \mathbf{v}, t) d_3 v. \tag{2}$$

Expression (1) holds only at equilibrium, just as entropy is defined only at equilibrium: however, this expression supplies an extension of this state function to non-equilibrium conditions, as *H* is the kinetic analog of entropy.

---

*Corresponding author. Tel.: +390516441711; fax: +390516441747.
E-mail address: domiziano.mostacci@unibo.it (D. Mostacci).

0378-4371/$ - see front matter © 2007 Elsevier B.V. All rights reserved.
doi:10.1016/j.physa.2007.06.015

The aim of this work is to analyze how this "generalized entropy" evolves in the course of DD fusion reactions. This can be done in the framework of kinetic theory [4,5].

It is to be expected that the entropy will be influenced by the distribution functions. Entropy is studied usually in the framework of thermodynamics: there the distribution functions and all the related effects are completely neglected, and therefore this approach is not apt to describe systems away from equilibrium, and among them nuclear reactions.

Taking the kinetic approach, the first problem to solve is to find the time evolution of the distribution functions of the various species involved. The main purpose of this work is to show the role that the spectra of the reaction products play in calculating entropy. In the following sections the distribution functions for the several species will be derived, following which the variation of entropy will be calculated, and the trend presented for a representative case.

2. Physical setting and time evolution of densities

Consider a homogeneous gas of deuterium in which D–D reactions take place. The D–D reaction has two possible outcomes, with almost equal probability

$$\mathrm{D} + \mathrm{D} \rightarrow \mathrm{n} + \mathrm{He}^3 \quad \mathrm{D} + \mathrm{D} \rightarrow \mathrm{p} + \mathrm{H}^3. \tag{3}$$

In the following, it will be assumed for simplicity that the branching has exactly 50% probability.[1] Hence, there will be five hadron species present: deuterium, helium-3, protons, tritium and neutrons, plus the electrons. Reaction products and their initial energy are summarized in Table 1.

To simplify the problem as much as possible, consider a situation in which the heat produced in the reaction is removed continuously (say, by bremsstrahlung) so that temperature remains constant, say 1 keV, and in which D–D reactions are rare enough and/or the system is observed early enough that the number densities of reaction products are very much smaller than that of Deuterium. Under such conditions, we can regard the distribution function of Deuterium to remain essentially Maxwellian at a (constant and uniform) temperature T:

$$f_\mathrm{D}(v,t) = n_\mathrm{D}(t) \times \frac{4}{\sqrt{\pi}} \left(\frac{m_\mathrm{D}}{K_\mathrm{B}T}\right)^{3/2} v^2 \exp\left\{-\frac{m_\mathrm{D}v^2}{2K_\mathrm{B}T}\right\}. \tag{4}$$

An equation can be written for Deuterium number density n_D as follows (here $\langle\ \rangle_\mathrm{M}$ has the usual meaning of the average over the Maxwellian distribution, and σ_DD is the cross section for the D–D reactions)

$$\frac{\partial n_\mathrm{D}}{\partial t} = -2n_\mathrm{D}^2 \langle v\sigma_\mathrm{DD}\rangle_\mathrm{M} \tag{5}$$

with the initial condition $n_\mathrm{D}(t=0) = n_0$.

It is convenient to express the problem in dimensionless form; to this end, the following dimensionless variables are defined (where j stands, here and elsewhere, for either n or p or He^3 or H^3):

$$\tilde{n}_j = \frac{n_j}{n_0}; \quad \tilde{Q}_j = \frac{v_{j,0}Q_j}{n_0\nu_\mathrm{D}}; \quad \nu_\mathrm{D} = n_0\langle v\sigma_\mathrm{DD}\rangle_\mathrm{M};$$
$$\tau = \nu_\mathrm{D}t; \quad \tilde{v} = \frac{v}{v_{j,0}}; \quad \tilde{f}_j = \frac{f_j}{n_0}v_{j,0}. \tag{6}$$

Eq. (5) and its initial condition then become

$$\frac{\mathrm{d}\tilde{n}_\mathrm{D}}{\mathrm{d}\tau} = -2\tilde{n}_\mathrm{D}^2 \quad \tilde{n}_\mathrm{D}(0) = 1 \tag{7}$$

and this equation can be solved readily to give

$$\tilde{n}_\mathrm{D}(\tau) = \frac{1}{1+2\tau}. \tag{8}$$

---

[1] Introducing the exact branching ratios presents no conceptual difficulty, however it would render the algebra more cumbersome without any conceptual addition to the problem at hand.