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Physica A 385 (2007) 356–362
PHYSICA A
www.elsevier.com/locate/physa

A continuous model of human dynamics

C. Bedogne'*, G.J. Rodgers

Department of Mathematical Sciences, Brunel University, Uxbridge, Middlesex UB8 3PH, UK

Received 12 February 2007; received in revised form 4 June 2007
Available online 27 June 2007

Abstract

Ideas and tools from statistical physics have recently been applied to the investigation of human dynamics. The *timing* of human activities, in particular, has been studied both experimentally and analytically. Empirical data show that, in many different situations, the time interval separating two consecutive tasks executed by an individual follows a heavy-tailed probability distribution rather than Poisson statistics. To account for this data, human behaviour has been viewed as a decision-based queuing system where individuals select and execute tasks belonging to a finite list of items as an increasing function of a task priority parameter. It is then possible to obtain analytically the empirical result $P(\tau) \sim 1/\tau$, where $P(\tau)$ is the waiting time probability distribution.

Here a continuous model of human dynamics is introduced using instead an *infinite* queuing list. In contrast with the results obtained by other models in the finite case we find a waiting time distribution explicitly depending on the priority distribution density function ρ. The power-law scaling $P(\tau) \sim 1/\tau$ is then recovered when ρ is exponentially distributed.

Keywords: Human dynamics; Complex systems

PACS: 89.75.Da; 02.50 Le; 89.75.Efa

1. Introduction

In the last few years the attention of physicists has been drawn to the application of ideas from statistical mechanics to biological, economic and social systems. In the last decade the physics literature has witnessed an explosion of inter-disciplinary research papers devoted to complex networks, econophysics and punctuated equilibrium systems. Very recently the quantitative description of human behaviour has also been addressed as a major question of modern science [1–4].

As a first step for the analysis of more complex human features, the *timing* of human activity is now beginning to be investigated. Motivated by statistical data indicating unexpected correlations between the timing of human activities as diverse as answering email messages or treating patients in a hospital emergency department, analytical models of human dynamics have been introduced.

*Corresponding author.
E-mail address: cesare.bedogne@gmail.com (C. Bedogne').

0378-4371/$ - see front matter
doi:10.1016/j.physa.2007.06.025

The empirical work, up to now, has mostly focused on the statistical properties of the inter-event time, that is the time interval between two consecutive tasks performed by the same individual, and on the time a given task has to wait before being executed. Rather unexpectedly, in both cases, the timing of human activity seems to follow heavy-tailed rather than Poisson statistics. Both experimental data and analytical results display inter-event time and waiting time distributions that are well approximated by power-law distributions $\tau^{-\alpha}$. For example, the distribution of the time differences between consecutive emails sent by the same individual goes like $\tau^{-\alpha}$, where $\alpha \simeq 1$. A similar statistical analysis has been performed to investigate the correspondence patterns of Darwin, Einstein and Freud [3,5]. The response time (that is the time interval between the day a given letter was received and the day it was answered) obeys again a power-law $\tau^{-\alpha}$ but in this case the exponent switched to $\alpha \simeq 1.5$, possibly indicating a different universality class. Analogous statistical properties have been identified in economic transactions [6] such as stock trading [7,8], in web-browsing [3,9] and in the execution of print-job submissions [10].

The emergence of power-law distributions in apparently very diverse human activities suggests that a fundamental feature of human dynamics is being uncovered.

Since power-law scaling is often considered a sign of complexity [9], the heavy-tailed statistics also indicate a possible (and intriguing) connection between human dynamics and complex systems.

According to the experimental results described in Ref. [1–4] the inter-event and the waiting time distributions exhibit a bursty behaviour where long periods of inactivity separate shorter periods of intense activity. Although the mechanism responsible for both bursts and heavy tails has not yet been clearly identified, it has been argued [1] that a wide range of human activities can be described as a decision-based queuing process where an individual executes tasks from a finite list of items according to some pre-defined selection protocol.

Previous models of queuing systems normally introduced a first-in-first-out protocol (in which tasks are executed in the order they appear in the list) or a random selection protocol. Both methodologies typically yield exponential distributions, therefore forbidding long waiting times, in contrast with the empirical data discussed above.

Assuming instead that humans select and execute tasks as an increasing function of a priority parameter assigned to them, it is possible to account for the empirical work, obtaining power-law distributed waiting times [1–3].

A priority selection protocol naturally emerges in many real-world situations, for example, when answering email messages of different importance or in the waiting room of an emergency hospital department [11], where the most urgent patients should obviously be treated first.

In the analytical models of human dynamics introduced in Ref. [1–4], tasks are thus assigned a priority index $x_i, i = 1, \ldots, L$ chosen from a distribution density function $\rho(x)$. At each time step a task is executed, removed from the list and replaced by a new task, whose priority is also chosen from $\rho(x)$. In contrast with the first-in-first-out and random choice protocols, tasks are now chosen stochastically as an increasing function of their priority. It is assumed that the probability to execute a task with priority x is $\Pi(x) \sim x^{\gamma}$. The parameter γ can then be tuned to interpolate between the random choice limit ($\gamma \to 0$) and the deterministic limit ($\gamma \to \infty$) where the highest priority task is always executed first. This model [1] yields the waiting time distribution

$$P(\tau) \sim \rho(\tau^{-1/\gamma})/\tau^{1+1/\gamma}. \tag{1}$$

In the deterministic limit the power-law $P(\tau) \sim 1/\tau$ is thus recovered, independent of the choice of the probability density function $\rho(x)$. In the random choice limit, the model instead yields an exponential distribution.

While these results [1] rely on heuristic arguments, requiring a mean field approximation, a similar model has been solved exactly [2] in the case $L = 2$ (where L is the total number of tasks in the waiting list), and its solution seems to confirm the conjectures developed in Ref. [1].

Here we introduce a model where the tasks to be executed are chosen from an infinite set treating both time and priority as continuous variables. We introduce a priority selection protocol and describe the queuing system with a integro-differential equation for the priority probability distribution of the tasks waiting to be executed (referring to the hospital situation described above, we will call it the "waiting room" priority probability distribution). We will observe, in particular, that the integro-differential equation admits a