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From unweighted to weighted networks with local information

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Abstract

In this paper, we analyze an evolving model with local information which can generate a class of networks by choosing different values of the parameter p. The model introduced exhibits the transition from unweighted networks to weighted networks because the distribution of the edge weight can be widely tuned. With the increase in the local information, the degree correlation of the network transforms from assortative to disassortative. We also study the distribution of the degree, strength and edge weight, which all show crossover between exponential and scale-free. Finally, an application of the proposed model to the study of the synchronization is considered. It is concluded that the synchronizability is enhanced when the heterogeneity of the edge weight is reduced.

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1. Introduction

Most recently, many natural and man-made systems [1–4] are described by complex networks, such as Internet, the worldwide airport networks (WAN) and cellular networks. These different complex networks share the common topological organization [1,5,6]. In particular, small-world properties [1] and scale-free behavior [3] have been observed in many real-life networks.

In this perspective, a wide array of models have been constructed to characterize the features of real-world systems [7–11]. The original algorithm that captured scale-free properties was proposed by Barabasi–Albert (BA model) in 1999 [3]. The local-world (LW) model [8] is another interesting model evolving unweighted networks only through local information. Barrat et al. [12]. have presented a model (BBV) which yields scale-free behavior for weight, strength and degree distributions. To our knowledge, all the previous models [8,12,14–17] can only generate topological networks or weighted networks, rarely both of them. Some problems arise here are how the variation in weight affects the characterizations of the networks, and what is

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the distinction between unweighted and weighted networks. Our model provides a beneficial tool for studying these issues. Meanwhile, when a node joins the network, it is impossible to know the global information in the network [8]. The new node connecting with the other old nodes only through local information is more practical.

In this paper, we propose an evolving network model with local information that shows the transition from unweighted to weighted. The distribution of the degree, node strength and edge weight can be widely tuned. While, with the increase in local information, the degree correlation of the network transforms from assortative to disassortative [13]. We finally analyze the synchronization of the generated networks. Compared with the previous models [14–17], the feature of our model is that the exponent of the edge weight can be tuned widely, ranging from 2 to ∞, and one can study the effect of weight on the dynamics of weighted networks based on the model.

The topological properties of a network are usually described by the adjacency matrix $\{a_{ij}\}$ (with $i,j = 1,\ldots,N$, where N is the size of the network), whose elements are 1 if an edge exists between node i and node j, or 0 otherwise [8–12]. The degree k_i of a node i is the number of its neighbors and is defined as $k_i = \sum_j a_{ij}$. Similarly, a weighted network is denoted by a matrix $\{w_{ij}\}$, which represents the weight on the edge connecting the nodes i and j. The strength s_i of a node i is defined as $s_i = \sum_{j\in\Gamma_i} w_{ij}$, where the sum runs over the neighbor node set Γ_i of node i. The strength of a node reflects the information about its connectivity and the weights of its edges [12]. In particular, the strength of a node is equal to the degree of a node if the network is defined as unweighted. In the following we only consider the undirected graph in which weight $w_{ij} = w_{ji}$.

The rest of the paper is structured as follows: In Section 2, the LW model is described. In Section 3, we present our model. In Section 4, the analytical calculation and simulation results are provided. In Section 5, the synchronization of the generated networks is analyzed. Finally, some conclusions are discussed in Section 6.

2. The local-world model

The LW model [8] is generated in the following way:

(1) Determining LW: the network starts from a few number of nodes (m_0) and (n_0) edges. Randomly choose M nodes from the existing network which are considered as LW of the new coming node at each time step.
(2) Topological growth: the new node n connects to m different nodes in its LW determined in step (1), where the nodes are preferentially chosen with the probability $\Pi_{local}(n \to i)$,

$$\Pi_{local}(n \to i) = \Pi'(i \in local{-}world)\frac{k_i}{\sum_{j,local} k_j}, \tag{1}$$

where $\Pi'(i \in local{-}world) = M/(t + m_0)$.

After t time steps, the procedure results in a network with $N = t + m_0$ nodes and $n_0 + mt$ edges.

3. Construction of our model

Based on LW model [8], Pan et al. [18] proposed two generalized local-world models (GLW) for weighted complex networks. Inspired by GLW models [18], we propose a simple model which evolves with the following mechanisms.

(i) Initialization: start from an initial seed of $N0$ nodes and $E0$ edges with assigned weight $w_0 = 1$. The following steps will be performed at each time step.
(ii) Determination of an LW: randomly select M nodes from the existing network, referred to as the LW of the new coming node.