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Theory and simulation for jamming transitions induced by a slow vehicle in traffic flow

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Abstract

We study the jamming transitions induced by a slow vehicle in a single-lane vehicular traffic. We use the dynamic model in which the normal vehicle allows to pass the slow vehicle just behind it with a probability. The fundamental diagram (flow density) changes highly by introducing a slow vehicle on a single-lane roadway. The spatio-temporal patterns are shown for the distinct traffic states. The dynamical state of traffic changes with increasing density. It is found that there are the three distinct states for the single-lane traffic flow including a slow vehicle. It is shown that the dynamical transitions among the distinct states occur at two values of density. The jamming transitions are analyzed theoretically. The transition points and fundamental diagram obtained by the theory agree with the simulation result.

Keywords: Traffic dynamics; Jamming transition; Phase diagram; Traffic state

1. Introduction

Transportation problems have attracted considerable attention in the field of physics [1–5]. Traffic flow is a kind of self-driven many-particle system of strongly interacting vehicles. Traffic flow has been studied by various traffic models: car-following models, cellular automaton (CA) models, gas kinetic models, and hydrodynamic models [6–23]. Traffic jams are typical signature of the complex behavior of traffic flow. Recent studies reveal physical phenomena such as the dynamical phase transitions and the nonlinear waves [1–5]. The jamming transition of spontaneous (phantom) jam is very similar to the conventional phase transitions and critical phenomena even if the traffic flow is a nonequilibrium system. While the jam induced by a bottleneck is not similar to the conventional phase transition but is the dynamical phase transition depending on the parameters of system dynamics [1–3].

Mobility is nowadays one of the most significant ingredients of a modern society. The bus (slow vehicle) often induces traffic jams because the bus moves slowly. The structure and formation of traffic jams in the two-lane highway have been studied by simulation when the bus prevents normal vehicles from moving fast in the first lane and the normal vehicles overtake the bus by changing the lane [24]. It has been shown that a

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jamming transition occurs when the density is higher than the critical point and a jam occurs just behind the bus. However, it has been hard to analyze the jamming transitions theoretically because the two-lane traffic flow is complex.

In this paper, we present a simple single-lane model to study the jamming transitions induced by the slow vehicle. We extend the optimal velocity model to take into account passing over the slow vehicle. We perform the simulation for the jamming transitions induced by a slow vehicle. We present the fundamental diagram in the single-lane traffic including a slow vehicle and allowing to pass the slow vehicle. We analyze the jamming transition induced by the slow vehicle analytically. We compare the simulation result with the theoretical one.

2. Model

We consider the traffic of vehicles flowing on the single-lane roadway. A slow vehicle is introduced into the single-lane traffic flow of normal vehicles. Normal and slow vehicles move on the single-lane roadway under periodic boundary condition. The maximal velocity of normal vehicles is always higher than that of the slow vehicle. We assume that the normal vehicle just behind the slow vehicle overtakes the slow vehicle sometimes.

Generally, a normal vehicle passes the slow vehicle by changing the lane. The passing on a two-lane roadway is modeled on a single-lane roadway as follows. The passing by lane changing is mimicked by exchanging the slow vehicle with the normal vehicle just behind the slow vehicle on the single-lane roadway. Analyzing the traffic flow analytically is possible by the modeling. We assume that the exchanging between the slow and normal vehicles occurs after a time or with probability p.

Except for the exchanging, vehicles move forward. We apply the optimal velocity model to the forward movement. The optimal velocity model is described by the following equation of motion of vehicle i:

$$\frac{\mathrm{d}^2 x_i}{\mathrm{d}t^2} = a\left\{ V(\Delta x_i) - \frac{\mathrm{d}x_i}{\mathrm{d}t} \right\}, \tag{1}$$

where $V(\Delta x_i)$ is the optimal velocity, $x_i(t)$ is the position of vehicle i at time t, $\Delta x_i(t)(= x_{i+1}(t) - x_i(t))$ is the headway of vehicle i at time t, and a is the sensitivity (the inverse of the delay time).

A driver adjusts the speed to approach the optimal velocity determined by the observed headway. The sensitivity a allows for the time lag $\tau = 1/a$ that it takes the speed to reach the optimal velocity when the traffic is varying. Generally, it is necessary that the optimal velocity function has the following properties: it is a monotonically increasing function and it has an upper bound (maximal velocity). The optimal velocity of normal vehicles has been given by

$$V(\Delta x_i) = \frac{v_{\max}}{2}[\tanh(\Delta x_i - x_s) + \tanh(x_s)], \tag{2}$$

where $v_{\max}$ is the maximal velocity of normal vehicles and x_s is the safety distance of normal vehicles. The slow vehicle is also given by the same optimal velocity function but the different maximal velocity

$$V(\Delta x_i) = \frac{v_{p\max}}{2}[\tanh(\Delta x_i - x_s) + \tanh(x_s)], \tag{3}$$

where $v_{p\max}$ is the maximal velocity of slow vehicle. We assume that the maximal velocity of the slow vehicle is less than that of normal vehicles.

3. Simulation result

We perform computer simulation for the single-lane traffic model including a single slow vehicle. We solve numerically Eq. (1) with optimal velocity functions (2) and (3) by using fourth-order Runge–Kutta method where the time interval is $\Delta t = \frac{1}{128}$. The slow vehicle exchanges with the vehicle just behind the slow vehicle every prescribed time. Thus, we simulate the traffic flow to allow passing the slow vehicle under the periodic boundary condition. The simulation is performed until the traffic flow results in a steady state.

We carry out simulation by varying density for various exchanging time period where safety distance $x_s = 4.0$, maximal velocity $v_{\max} = 2.0$ of normal vehicles and sensitivity $a = 2.0$. We set the length of roadway as $L = 1050$. We study the nonoscillating traffic flow for $a \geqslant 2.0$ [1].