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Pinning control of complex dynamical networks with general topology

L.Y. Xiang*, Z.X. Liu, Z.Q. Chen, F. Chen, Z.Z. Yuan

Department of Automation, Nankai University, Tianjin 300071, PR China

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Abstract

Recently, the researches on pinning control of complex dynamical networks have mainly focused on such networks with very specific coupling schemes (e.g., symmetric coupling, uniform coupling and linear coupling). However, most real networks often consist of local units, which interact with each other via asymmetric and heterogeneous connections. In this paper, pinning control of a continuous-time complex dynamical network with general coupling topologies is studied. Some generic stability criteria based on master stability function (MSF) are derived for such a general controlled network, which guarantee that the whole network can be pinned to its equilibrium by placing feedback control only on a small fraction of nodes. Then, these results are extended to discrete-time case. Previous results about symmetric, uniform or linear coupled networks in this area are included as special cases of the present work. Numerical simulations of directed networks with weighted coupling pinned by specifically selective pinning scheme are given for illustration and verification.

Keywords: Complex dynamical network; Directed graph; Master stability function; Pinning control; Transverse Lyapunov exponents (TLEs); Weighted networks

1. Introduction

We live in a world of networks. In fact any complex system in nature and societies can be modeled as a network, where vertices are the elements of the system and edges represent the interactions between them. The last decade has witnessed the birth of a new movement of interest and research in the study of complex networks [1–5], which is pervading all kinds of sciences today, ranging from physical to chemical, biological, information technology, mathematical, and even to social sciences. In particular, the discovery of some significant characteristics of complex networks, such as the now-well-known small-world effect and scale-free feature, has led to dramatic advances in this active research area. Its impact on modern engineering and technology is prominent and far-reaching.

Recently, the interplay between the complexity of the overall topology and the collective dynamics of complex networks gives rise to a host of interesting effects. Especially, there are attempts to control the

*Corresponding author.
E-mail address: xlyzhjl1980@mail.nankai.edu.cn (L.Y. Xiang).

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dynamics of a complex network and guide it to a desired state such as an equilibrium point or a periodic orbit of the network. A basic assumption of previous work [6–9] is that the local units are coupled symmetrically or linearly with the same coupling strength. However, in many circumstances this simplification does not match satisfactorily the peculiarities of real networks. For instance, the WWW [10,11], metabolic networks and citation networks [12,13] are all directed graphs, whose coupling matrices are asymmetric. In addition, some phenomena such as the diversity of the predator–prey interactions in food webs [14,15], different capabilities of transmitting electric signals in neural networks [16,17], unequal traffic on the Internet [18] or of the passengers in airline networks [19,20] explain the existence of weighted wirings [21]. Moreover, the inner coupling may be nonlinear [22]. Therefore, research on such general complex dynamical networks in the presence of asymmetric and heterogeneous connections is of particular importance and has been an immensely challenging undertaking in the current literature.

To our knowledge, the important pinning control problem of complex dynamical networks with asymmetric and heterogeneous connections has not been deeply investigated, and this is the central focus of the present paper. The main contribution of this paper is to develop a general approach to stabilize such a network onto some desired homogeneous stationary states by injecting only a small number of local feedback controllers when these states are unstable without control. Some simple feedback controllers are designed and some generic stability criteria yielding explicit constrains on the overall coupling strengths are derived based on the ideas of MSF, which is introduced to synchronized coupled systems with general coupling topologies in [23–30]. Here, this function defines a region of stability, in which the network is controllable, in terms of the eigenvalues of the coupling matrix and the feedback gain matrix. Some numerical simulations for verifying the theoretical results are given in Section 4. Finally, Section 5 concludes the investigation.

2. Installation model description and stability analysis

Let us consider a new yet generic complex dynamical network consisting of N identical nodes with fairly general coupling topologies, where each node is an m-dimensional dynamical system, described by

$$\dot{x}_i = f(x_i) - \frac{a}{k_i^{\beta_w}} \sum_{j=1}^{N} L_{ij} h(x_j), \quad i = 1, 2, \ldots, N, \tag{1}$$

where $x_i = (x_{i1}, x_{i2}, \ldots, x_{im})^T \in R^m$ represents the state vector of the ith node, and the function $f(\cdot)$, describing the local dynamics of the nodes, is continuously differentiable and capable of producing various rich dynamical behaviors, including fixed points, periodic orbits and chaotic states. The overall coupling strength a is assumed positive; k_i is the out-degree of node i and β_w is a tunable weight parameter. Also, $h(\cdot)$ is an arbitrary but continuously differentiable function of each node's variables that is used in the inner-coupling, while the real matrix $L = (L_{ij})_{i,j=1}^N$ is the usual (symmetric) Laplacian matrix with diagonal entries $L_{ii} = k_i$ and off-diagonal entries $L_{ij} = -1$ if node i and j are connected by a link, and $L_{ij} = 0$ otherwise.

Clearly, many coupling schemes are included in Eq. (1). For example, if the isolated node is a Chen system and the inner coupling is through the Chen "x_1" and "x_2" components, then the function $h(x)$ is just linear as follows:

$$h(x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} x.$$

Furthermore, the parameter $\beta_w = 0$ recovers that the network is unweighted and undirected, and the condition $\beta_w \neq 0$ corresponds to a network with not only weighted but also directed configuration. It should be noted that this is a special kind of directed network, where the number of *in*-links is equal to the number of *out*-links in each node and the directions are encoded in the strengths of in and out-links [33].

For convenient analysis, we let:

$$b_{ij} = -L_{ij}/k_i^{\beta_w}, \tag{2}$$