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Compactons and solitary patterns solutions to fifth-order KdV-like equations

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Abstract

New ansatze that involve hyperbolic functions are introduced to handle modified fifth-order KdV equations. A set of entirely new compactons, solitary patterns and periodic solutions is constructed. The study introduces new approaches and presents useful guidance for other related problems.

Keywords: Fifth-order KdV equation; Hyperbolic functions ansatze; Compactons

1. Introduction

In solitary wave theory, solitons are created as a result of the balance between the convection term uu_x and the dispersion term u_{xxx} in the pioneering nonlinear dispersive Korteweg de-Vries equation

$$u_t + auu_x + u_{xxx} = 0. \tag{1}$$

Solitons are localized waves that propagate without change of its identities and stable against mutual collisions. There are many types of solitary waves that are of particular interest. One type is the *solitary waves* which are localized travelling waves approaches zero at large distances. The *kink waves* is another type which rise or descend from one asymptotic state to another. The *peakons* are peaked solitary wave solutions or peak-shaped interacting waves that have a finite jump in the first derivative of the solution $u(x, t)$. Peakons arise from the Camassa–Holm equation and the Degasperis–Processi equation. *Cuspons* are also solitary waves where the derivative of $u(x, t)$ at the jump diverges. Recently, a new type, named *compactons*, that make a qualitative change in the physical structures of the solutions, has arouse. Compactons are solitons with compact spatial support such that each compacton is a soliton confined to a finite core.

It is the objective of this work to further derive sets of new travelling wave solutions for two fifth-order Korteweg de-Vries-like equations (fKdV) given by

$$u_t + auu_x + buu_{3x} + u_{5x} = 0, \tag{2}$$

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and

$$u_t + au^2u_x + bu^2u_{3x} + u_{5x} = 0, \tag{3}$$

with constant parameters a and b and $u_{kx} = \partial^k/\partial x^k$. The general case that includes $u^n u_x$ and $u^n u_{3x}$ was investigated for $n \geqslant 2$ in Ref. [1] with distinct approaches and distinct physical structures solutions.

The generalized Korteweg de-Vries equations describe motions of long waves in shallow water under gravity and in a one-dimensional nonlinear lattice [1–10,18]. The nonlinear generalized KdV equation is an important mathematical model with wide applications in quantum mechanics and nonlinear optics. Typical examples are widely used in various fields such as solid state physics, plasma physics, fluid physics and quantum field theory [10–18].

A great deal of research work has been invested during the past decades for the study of the fKdV equation. The main goal of these studies was directed towards its analytical and numerical solutions. Several different approaches, such as Backland transformation, a bilinear form, and a Lax pair, have been used independently by which soliton and multi-soliton solutions are obtained. Ablowitz et al. [1] implemented the inverse scattering transform method to handle the nonlinear equations of physical significance where soliton solutions and rational solutions were developed. The tanh method, developed by Malfliet [5,6], is heavily used in the literature to handle nonlinear evolutions equations.

The objectives of this work are twofold. Firstly, we seek to introduce new ansatze that involve hyperbolic functions to establish new compacton, solitary patterns, and periodic solutions for (2) and for (3). Secondly, we aim to show that the new schemes provide useful guidance for related identical nonlinear problems.

In what follows, we highlight the main features of the proposed methods. The power of the methods, that will be used, is its ease of use to determine shock or solitary type of solutions.

2. The methods

We first unite the independent variables x and t into one wave variable $\xi = x - ct$ to carry out a PDE in two independent variables

$$P(u, u_t, u_x, u_{xx}, u_{xxx}, \ldots) = 0, \tag{4}$$

into an ODE

$$Q(u, u', u'', u''', \ldots) = 0. \tag{5}$$

Eq. (5) is then integrated as long as all terms contain derivatives. Usually the integration constants are considered to be zeros in view of the localized solutions. However, the non-zero constants can be used and handled as well.

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2.1. A rational sinh–cosh ansatz

As formally presented in Refs. [4,10,18] we introduce a rational cosh ansatz

$$u(x,t) = \left\{ \frac{\alpha \cosh^2[\mu(x - ct)]}{1 + \lambda \cosh^2[\mu(x - ct)]} \right\}^{1/n}, \quad n = 1, 2 \tag{6}$$

and a rational sinh ansatz

$$u(x,t) = \left\{ \frac{\alpha \sinh^2[\mu(x - ct)]}{1 + \lambda \sinh^2[\mu(x - ct)]} \right\}^{1/n}, \quad n = 1, 2. \tag{7}$$

The ansatze (6) and (7) were successfully used in Refs. [2–4].