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Physica A 371 (2006) 667–673
PHYSICA A
www.elsevier.com/locate/physa

Density viscous continuum traffic flow model

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Received 21 December 2005; received in revised form 13 January 2006
Available online 27 April 2006

Abstract

A new traffic flow model called density viscous continuum model is developed to describe traffic more reasonably. The two delay time scales are taken into consideration, differing from the model proposed by Xue and Dai [Phys. Rev. E 68 (2003) 066123]. Moreover the relative density is added to the motion equation from which the viscous term can be derived, so we obtain the macroscopic continuum model from microscopic car following model successfully. The condition for stable traffic flow is derived. Nonlinear analysis shows that the density fluctuation in traffic flow induces density waves. Near the onset of instability, a small disturbance could lead to solitons determined by the Korteweg-de-Vries (KdV) equation, and the soliton solution is derived. The results show that local cluster effects can be obtained from the new model and are consistent with the diverse nonlinear dynamical phenomena observed in the freeway traffic.

Keywords: Traffic flow; Continuum hydrodynamic model; Viscous

1. Introduction

Phenomena generated by traffic flow are surprisingly rich and colorful, so the models that attempt to describe the system are numerous and multi-faceted, each aiming to explaining certain aspects of the system. Since the pioneering work by Lighthill and Whitham [1,2] and Richards [3] on kinematic waves of vehicular traffic flow, many efforts were devoted to improving the Lighthill–Whitham–Richards (LWR) theory through developing higher-order continuum models. In 1971, Payne [4] introduced a higher-order continuum traffic flow model including a dynamic equation, which is derived from the Newell [5] car following theory. It claimed that the velocity of traffic flow will reach equilibrium state with the delay time τ, and the velocity at the location (x, t) is determined by the traffic density there $(x + \Delta x, t)$, that is,

$$v(x, t + \tau) = V(\rho(x + \Delta x, t)). \tag{1}$$

Using the Taylor expansion to the above equation leads to

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{\mu}{\rho\tau}\frac{\partial \rho}{\partial x} + \frac{V(\rho) - v}{\tau}, \tag{2}$$

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doi:10.1016/j.physa.2006.03.034

where $V(\rho)$ is the optimal velocity function, and $\mu = -0.5\partial V/\partial\rho$ is the anticipation coefficient. The first term on the right-hand side is called the anticipation term, which reflects a driver's reaction to the car in front of him/her. The second term represents a relaxation to equilibrium state. The Payne model improved the LWR model by incorporating the motion equation and taking into account the acceleration and inertial effects. The model could describe the amplification of small disturbances in heavy traffic and allow fluctuations of speed around the equilibrium state. The viscous model of traffic flow bears a strong resemblance to the Navier–Stokes equations of compressible fluid in one space dimension and much work is related to it [6–8], from which some main results could be concluded, such as the smoothing of the shocks in the PW model [4,9] owing to the viscosity term in the Kühne's viscous model [6]. Vehicle density could fluctuate for various reasons to form a density wave in traffic flow, which leads to the traffic jams. Different nonlinear wave equations have been derived to describe the corresponding density waves, among which the Burgers equation, Korteweg-de-Vries (KdV) equation and modified Korteweg-de-Vries (mKdV) equation depict the density waves appearing in the distinct regions, respectively. A number of research work has been done in this aspect for the car following model [10–12]. In 2000, Berg et al. [13] developed a systematic method for relating the car following model to continuum models for road traffic. According to this relation, the density waves in car following models were studied from another viewpoint [14,15]. Recently, based on Jiang's model [16], Gupta and Katiyar [17] derived a new viscous model by using the headway–density relationship presented by Berg. The certain qualitative properties were investigated through linear analysis and numerical simulation. However, for continuum model, less investigation has been reported about the nonlinear analysis. Kurtze and Hong [18] derived the KdV equation from the hydrodynamic model and showed that the traffic soliton appears near the neutral stability line.

As it is known that viscosity in fluids arise from resistance to change of fluid motion and smoothes drastic changes in density, speed and pressure. One might suspect that viscosity, in traffic flow model, is the drivers' tendency to resist sharp changes in speed, which may lead to viscous terms in traffic speed dynamics [19]. We shall develop a mechanics model with a viscous term in the motion equation and two delay time scales for studying the qualitative properties of density wave.

2. Model

First, we give the process of modelling. In the light of two delay time scales model [20], the velocity–density relation is

$$v(x + v\tau, t + \tau) = V_e(\rho(x + vT, t + T)). \tag{3}$$

We agree that the determination of velocity v accomplished in the relaxation time τ, in the mean time the process is completed by an adjustment with driver reaction time T to the anticipated velocity V_e. The relaxation time τ includes the drivers' reaction time T as well as mechanical delay of the vehicles. But it must be noticed that the velocity at the location (x, t) is determined by the traffic density at the location $(x + \Delta x, t)$, which is in agreement with the idea of Payne. So we give the relation as

$$v(x + v\tau, t + \tau) = V_e(\rho(x + \Delta x + vT, t + T)). \tag{4}$$

In the car following model, it is reasonable to include the effects of relative velocity because in real traffic drivers' reactions are affected not only by the distance of two successive vehicles, but also by the relative velocity between them [21]. Such a car following behavior, when being considered into a continuum traffic flow, gives rise to the viscosity model. From the viewpoint of the macroscopic model, we think the relative density is more reasonable. We deduce the following relation:

$$v(x + v\tau, t + \tau) = V_e(\rho(x + \Delta x + vT, t + T)) + \mu(\rho(x + \Delta x, t) - \rho(x, t)), \tag{5}$$

where $\mu = b_0 V_e'(\rho) < 0$, b_0 is a positive constant. Then we make the Taylor series expansion on both sides of Eq. (5) around (x, t) and $\rho(x, t)$, and neglect the higher-order terms to get an approximation as

$$v + v\tau v_x + \tau v_t = V_e(\rho) + V_e'(\rho)[\rho_x(\Delta x + vT) + \rho_t T] + \mu\rho_x\Delta x + \mu\rho_{xx}\Delta x^2, \tag{6}$$