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Classical dissipation and asymptotic equilibrium via interaction with chaotic systems

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Abstract

We study the energy flow between a one-dimensional oscillator and a chaotic system with two degrees of freedom in the weak coupling limit. The oscillator's observables are averaged over an initially microcanonical ensemble of trajectories of the chaotic system, which plays the role of an environment for the oscillator. We show numerically that the oscillator's average energy exhibits irreversible dynamics and 'thermal' equilibrium at long times. We use linear response theory to describe the dynamics at short times and we derive a condition for the absorption or dissipation of energy by the oscillator from the chaotic system. The equilibrium properties at long times, including the average equilibrium energies and the energy distributions, are explained with the help of statistical arguments. We also check that the concept of temperature defined in terms of the 'volume entropy' agrees very well with these energy distributions.

Keywords: Low-dimensional chaos; Dissipation; Energy flow; Brownian motion

1. Introduction

Low-dimensional chaotic systems can, under appropriate circumstances, play the role of thermodynamical heat baths [1–7]. If a slow system with few degrees of freedom is weakly coupled to a fast chaotic system, the slow system's average trajectory can dissipate energy into the chaotic one at short times.

One of the initial motivations for the consideration of low-dimensional chaotic dynamics as environments for macroscopic systems was the work by Brown et al. [8] on the ergodic adiabatic invariant. For a chaotic Hamiltonian system with a slowly varying parameter, the volume of the energy shell is the only adiabatic invariant. Brown et al. showed that the first order correction to this invariant has a diffusive temporal behavior. Besides, if such time varying parameter is thought of as a second system coupled to the chaotic one, then this diffusive correction of the adiabatic invariant would lead to a dissipative force on the slow system [1].

This problem was again reformulated by Berry and Robbins [2] in terms of a system of interest interacting with an environment, or 'thermal bath'. The average force acting on the system of interest due to the coupling with the chaotic system can be calculated in adiabatic approximation. The lowest order part of this force was

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shown to be the Born–Oppenheimer force and the next order to be a geometric magnetism type of force plus a deterministic friction—a force proportional to the slow system's velocity. Friction is therefore generated in the context of small systems, without the need for the thermodynamical limit, and the chaotic nature of the motion becomes the essential ingredient. Indeed, typical correlation functions of chaotic systems decay exponentially with the time, as opposed to the quasi-periodic behavior observed in regular systems. Jarzynski [4] showed later that coupling to a low-dimensional chaotic motion can also lead to a 'thermalization' of the system of interest, very much like the thermalization of a Brownian particle interacting with a large thermal bath. Finally, de Carvalho and de Aguiar [3] established a connection between de formalism developed by Caldeira and Leggett [9] for describing dissipation via coupling with a thermal bath and via coupling with a chaotic system.

In this paper we revisit this problem from the classical point of view and consider specific examples of two-dimensional chaotic systems coupled to a one-dimensional oscillator. Our main purpose here is to understand the energy flow between the system of interest, which we shall call 'the oscillator' and the chaotic system to which this oscillator is coupled. We study the energy flow at short times and the approach to equilibrium at long times. In order to consider the chaotic system as playing the role of an environment, we assume that the only information available from this system is its initial energy. For the oscillator this implies that only microcanonical averages of its observables (over the chaotic system variables) are accessible. Therefore, a typical numerical calculation corresponds to fix an initial condition for the oscillator and to evolve an ensemble of trajectories whose initial conditions for the chaotic variables are randomly selected at a fixed (chaotic system) energy shell.

For short times our numerical simulations show that the average energy of the oscillator may increase or decrease, absorbing energy from the chaotic system or dissipating energy into the chaotic system. The initial energies of both systems are what ultimately dictates which of the two possibilities actually occurs. In particular, there exists initial values of these energies such that no exchange occurs on the average. We use linear response theory to study the energy flow in the short time limit. We show that the average motion of the oscillator follows a Langevin type of equation with frequency-dependent friction and a quadratic correction to the oscillator potential, similar to the Born–Oppenheimer force that appears in the adiabatic theory. We also derive a simple condition for dissipation or absorption of energy by the oscillator involving the ratio of the initial energies of the systems. This theoretical prediction agrees well with our numerical calculations for short times, but it is not accurate to predict the long time behavior.

Our simulations show that, at long times, the average energy of both the oscillator and the chaotic system tend to an equilibrium. The value of these equilibrium energies depends once again on the initial conditions. The connection between asymptotic thermalization and initial conditions is well known for a Brownian particle. In that case, the increase or decrease of the average energy of the particle depends on its initial energy E_0 and on the temperature T of the thermal bath. The particle absorbs energy from the reservoir if $E_0 < k_B T$ and loses energy into it if $E_0 > k_B T$, thermalizing always at $k_B T$. Here we have a similar situation, with the increasing or decreasing of the average energy of the oscillator depending only on its initial energy and on the initial energy of the chaotic system. However, contrary to the case of the Brownian particle, the condition for equilibrium at long times is generally different from the condition of no energy exchange at short times. Despite the theoretical work of Jarzynski [4] on the long term thermalization of these systems, it is still not clear how the asymptotic states depend on the initial conditions of both sub-systems. The energy distribution of the sub-systems at equilibrium is also an important open issue. These distributions are not likely to be of the Boltzmann type, since the systems are small, and they may depend not only on the initial conditions but also on the density of states of the systems involved [10–12]. In this paper we shall derive the long time equilibrium conditions and energy distributions explicitly for two model systems. Finally, we use the definition of temperature proposed in Ref. [13] for small systems and check that it agrees completely with the statistical calculation in terms of the density of states and energy distributions.

We emphasize that our approach uses the linear response theory, and no explicit assumptions on adiabatic properties of the oscillator is required. Despite the difficulties involved in the calculation of the response function for microcanonical ensembles, the formulation of this problem in terms of linear response theory is of great interest for the study of its quantum analog [7]. We recall that the usual formulation of quantum dissipation [9] involves response functions and that the adiabatic approximation leads to frustrating results in