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Phase-synchronisation in continuous flow models of production networks

Bernd Scholz-Reiter, Jan Topi Tervo*, Michael Freitag

Bremen Institute of Industrial Technology and Applied Work Science, Hochschulring 20, D-28359 Bremen, Germany

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Abstract

To improve their position at the market, many companies concentrate on their core competences and hence cooperate with suppliers and distributors. Thus, between many independent companies strong linkages develop and production and logistics networks emerge. These networks are characterised by permanently increasing complexity, and are nowadays forced to adapt to dynamically changing markets. This factor complicates an enterprise-spreading production planning and control enormously. Therefore, a continuous flow model for production networks will be derived regarding these special logistic problems. Furthermore, phase-synchronisation effects will be presented and their dependencies to the set of network parameters will be investigated.

Keywords: Production network; Continuous flow model; Synchronisation

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1. Introduction

Production networks are distinguished by a permanently growing complexity and are nowadays more than ever forced to adapt fast to dynamically changing markets. These factors complicate an enterprise-spreading production planning and control enormously. Therefore, recent studies of production networks were focussed on the dynamical aspects and it has been discovered that the material transport within those networks can be considered as a physical transport problem (e.g., [1,2]) with balance equations for delivered material, as already mentioned in Ref. [3]. In particular, non-linear behaviour in production systems was investigated [4] and models were found to exhibit complex, oscillatory and even chaotic behaviour [5–7]. Thus, stability analyses of different models and topologies were performed [8,9] to detect critical sets of parameters and stabilising influences. This is in fact very useful for the supression of the bullwhip effect, that describes the amplification of oscillatory amplitudes of delivery rates along the supply chain [10]. But another very interesting and sophisticated field of non-linear dynamics, the phenomenon of synchronisation, has not yet been applied to production systems. It should be investigated, if synchronisation also had such a stabilising effect on the network or helped to avoid the bullwhip effect. It is of further practical economic importance, if

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*Corresponding author.

E-mail addresses: bsr@biba.uni-bremen.de (B. Scholz-Reiter), ter@biba.uni-bremen.de (J.T. Tervo), fmt@biba.uni-bremen.de (M. Freitag).

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synchronisation can lead to a better adjustment of production processes within a supply chain or network and, thereby, help to improve the competitiveness of these enterprises.

One of the first reports on synchronisation was made in the 17th century by Huygens [11], who discovered that two pendulum clocks mounted on the same beam oscillated with the same frequency. Since then, synchronisation was found in many disciplines of physics. For instance, Rayleigh found nearby organ tubes to synchronise their frequencies [12] and van der Pol studied the synchronisation of electric circuits [13]. More recently, two semiconductor lasers were found to exhibit synchronisation of their light intensities [14] and for clusters of driven acoustic cavitation bubbles simulations also showed synchronisation effects [15]. These examples and numerical simulations (e.g., Ref. [16]) show that synchronisation is a well-defined and well-examined phenomenon for a large variety of physical applications. But synchronisation occuring in complex systems such as supply chains or supply nets are neither well investigated nor well understood. Hence, in this paper we will derive extensions to a continuous flow model for production networks introduced by Helbing [1], that is based on fluid-dynamics models (e.g., see Ref. [17]). The continuous flow approach has several advantages, but also disadvantages as well, compared to a discrete event approach. Since in production systems mostly discrete products have to be handled, a discrete event simulation (DES) model would be favourable here. But with continuous flow models consisting of a set of differential equations large networks can be modelled with less computational effort than with a DES model. Additionally, continuous flow models can take non-linear interactions into account and are suitable for online control under dynamically changing conditions [2,8].

In contrast to the model introduced by Helbing [1], we will on the one hand restrict the possible network topologies: many companies concentrate on their core competences and start cooperations with suppliers and distributors to improve their position at the market. The consequence for the model is, that only one product can be manufactured in a node. On the other hand, we will focus on existing logistic policies, namely the sealing-off to competitors. Consequently, the only information an enterprise can access to plan and control its production is the demand, respectively, the orders of customers. Based on this model, synchronisation phenomena will be investigated and especially the capability of parameters to change the coupling strength between the nodes will be identified. This coupling is basically bidirectional, but consists of different types in both directions. One is given by the flow of information, e.g., orders or buffer levels, and the other one by the flow of material, e.g., delivered goods or educts. Contrarily, classically coupled systems like Huygens clocks [11] basically only exhibit one kind of coupling, e.g., elastic forces.

2. Production network model

2.1. Basic topology

We will consider a production network consisting of N nodes $i,j \in \{1 \dots N\}$ manufacturing the products $p \in \{1 \dots N\}$. Every node is assumed to produce exactly one product due to its concentration on core competences. So the specification of a node j determines uniquely the product ($p = j$) that is produced there. The nodes are connected by edges which represent the coupling between the nodes. More precisely, the coupling is realised by the flow of material and information along these edges. Whereas, in this model no capacity constraints are implemented, only the delivery time T_{ij} from node i to node j is taken into consideration.

Every node i is composed of an output buffer O_i, where the manufactured goods are stored for delivery, and a production unit, characterised by the production rate Q_i. In order to ensure a continuous production, a safety stock for every incoming product (educt) is established. Thus, node i also contains input buffers I_i^j to store goods from nodes j. Fig. 1 shows the schematic illustration of a network with four nodes and depicts the construction of a single node i. The production process starts with the delivery of the educts from the input buffers to the production unit. Their ratios are given by the coefficient matrix c_i^j. On the other hand, the distribution of products from the output buffers to other nodes is described by the coefficient matrix d_j^i. Naturally, all these coefficients have to fulfil the conditions $0 \leqslant c_i^j \leqslant 1$, $0 \leqslant d_j^i \leqslant 1$ and $\sum_{j=1}^N c_i^j \leqslant 1$, $\sum_{i=1}^N d_j^i \leqslant 1$. To ensure the conservation of the flow, there must be an inflow of resources into the network to every node i, given by $c_i^0 = 1 - \sum_{j=1}^N c_i^j$, and an outflow of products to external consumers, given by $d_j^{N+1} = 1 - \sum_{i=1}^N d_j^i$.