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Kinks, logarithmic tails, and super-stability in bi-disperse granular media

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ABSTRACT

Determining the stability of granular matter piles is of basic concern in understanding many real-world phenomena (e.g. landslides, debris flow, and avalanches). While extensive literature exists dealing with the stability of mono-disperse systems, models for the dynamical behavior of poly-disperse media are still uncommon. Here, a simple experimental setup that probes the dependence of the repose angle (θ) for different proportions of granular mixtures is described. We demonstrate that a cellular automata (CA) grid with rules based on gravitational effects can phenomenologically mimic the dynamics of experimental data in terms of: (1) the presence of disruptions or kinks in an otherwise perfectly straight slope; (2) a concave logarithmic tail, indicative of the nature of the granular medium; and (3) the existence of supra-maximal repose angles for binary mixtures such that $\theta_{mix} > \theta_1, \theta_2$, which can lead to super-stability, or 90-degree slopes. This latter result has profound implications because of the ubiquity of vertical slopes in nature, while standard continuum approaches cannot account for such (because it entails an infinite value of the coefficient of friction).

1. Introduction

Experience dictates that pouring liquid to a container, say water into a glass, will yield a horizontal topmost layer of minimum potential. Whatever ripples are formed by small perturbations tend to decay in time, leaving behind the same exact flatness. Doing the same for liquid-like granular materials will yield utterly different results. While it is true that granular materials are almost fluid-like in their ability to flow [1,2], the particulate nature of the individual granules come into play when pouring granular materials. Instead of a horizontal topmost surface, granular heaps tend to be inclined at an angle. Moreover, ripples, or kinks, tend to be permanent, due to small differences in the particle sizes [3,4]. Continuous addition of granular materials will eventually lead to a state when the heap can no longer be steeper. At this stage, the heap is said to be inclined at the maximum angle of stability θ_{max} [5]. When θ_{max} is reached, an avalanche event occurs wherein the heap collapses and the granules rolling off from the higher part of the slope produce a cascade effect on the lower ones. The angle that forms after an avalanche is referred to as the angle of repose θ — the easiest to measure and most straightforward manifestation of granular stability (GS).

Landslides, debris flows, mudslides, and avalanches, among others, are examples of processes in which our understanding of the dynamics is quite limited because the physics associated with GS is still unresolved. Recently, statistics of granular mound slides reveal reasonable similarities with those in actual landslide volume surveys, hinting at their possible use as scaled-down models of actual natural systems [6]. While the majority of the work done on GS is concerned with mono-disperse elements [7,8], most real systems require combinations of two or more granular materials in diverse proportions. Hence, a mathematical tool that describes GS for poly-disperse media is needed in order to provide a more realistic picture

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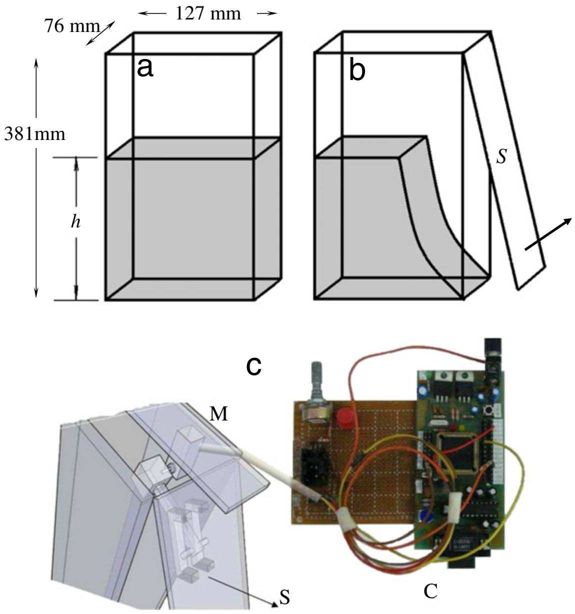

Fig. 1. Granular step setup. (a) Initial 90-degree slope (i.e. a ''step''), (b) slope formation by opening S, and (c) the motorized setup. A Tower Pro DC servo motor is employed in the 3D container to regulate the opening rate of S. The motor M is controlled by the programmed circuit (C) in such a way that (1) it will hold and contain the binary mixture within the container before opening S and (2) S swings quickly enough that it will not touch the mixture. A linear voltage supply [3.2, 5.4] V is provided for the power line and a pulse-width modulated square wave (amplitude [0, 3.3] V and ($f = 83.33$ Hz) is received by the motor control line.

for natural processes. However, previous works on binary mixtures often deal with the segregation of material components alone [1,8–10] and fail to hint at quantifiable parameters that account for GS.

Describing granular material is still a challenge owing to the fact that the collective behavior of an ensemble of numerous solid grains does not obey any known hydrodynamic equations [1]. The usual approach uses thermodynamics, where one normally ignores the nature of the medium and models the system as a continuum [11]. Continuum approaches, in general, ignore the underlying intra- and inter-particle interactions happening at the single-grain level that are important for the emergence of the fluid-like properties of granular matter [2,12].

Because of the inherent intricacy in poly-disperse granular media, discretized models are commonly used to provide a realistic description of their behavior. In particular, cellular automata (CA) based models where lattice cells in space interact via nearest-neighbor rules, provide a simple and efficient approach in treating particle-to-particle interactions. CA models have long been used to model granular systems, from the sandpile model of Bak et al. that looks into avalanche formations [13,14], to other extreme events (landslides, avalanches) [15] for large-scale systems and even for table-top granular piles [6]. Interesting properties of granular materials such as spontaneous stratification of binary mixtures have also been captured by CA models [16]. Recently, CA models of sandpile avalanches have been treated using the theory of self-organized branching process in demonstrating self-organized criticality in dissipative systems [17–19].

In this article, a simple setup to probe θ is utilized (see Fig. 1): *a rectangular container filled to some height h with granular material, that is opened rapidly at one of its sides*. We demonstrate that for binary mixtures of granular materials, a repose angle θ_{mix} that is greater than the angle of repose of its components θ_i ($\theta_{\text{mix}} > \theta_1, \theta_2$) can exist in our granular setup, in addition to the previous observations of Samadani and Kudrolli [20] and Pohlman et al. [21]. Further, we confirm the presence of kinks and logarithmic tails in bi-disperse piles that have been previously observed only for mono-disperse granular matter [3]. The experiments are complemented by a CA simulation in which the state evolves as dictated by the effective cell interaction, the slipping probability σ or σ^*. All of the dynamics described above, including the average value and fluctuations of θ as a result of the mixing ratio, are phenomenologically captured by our numerical experiments. The model is versatile to allow simulation of polydisperse mixtures with arbitrary components.

2. Granular step setup

Of the many types of setup used in θ-investigation, e.g. rotating tumblers [4,9], continuous addition of grains (which is the basis for the sandpile model [13,14]), and geological uplift used in some models [22], the collapsing granular step is deemed to be more realistic and practical in modeling natural slope formations found in nature [23,24]. A schematic of the granular setup used here is illustrated in Fig. 1. A rectangular container with width $w = 76$ cm and length $l = 127$ cm is filled with granular material up to height h, after which a passive wall S is opened to allow avalanche events to happen, leaving behind a heap inclined at θ. By setting up the heap to a 90-degree inclination (i.e. more than the maximum angle of