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Antiplane crack analysis of a functionally graded material by a BIEM

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Abstract

Elastostatic analysis of an antiplane crack in a functionally graded material (FGM) is performed by using a hypersingular boundary integral equation method (BIEM). An exponential law is applied to describe the spatial variation of the shear modulus of the FGM. A Galerkin method is applied for the numerical solution of the hypersingular traction BIE. Both unidirectional and bidirectional material gradations are investigated. Stress intensity factors for an infinite and linear elastic FGM containing a finite crack subjected to an antiplane crack-face loading are presented and discussed. The influences of the material gradients and the crack orientation on the stress intensity factors are analyzed.

Keywords: Antiplane crack analysis; Functionally graded materials; Boundary integral equation method; Stress intensity factors; Fracture mechanics

1. Introduction

Functionally graded materials (FGMs) are preferred and favoured in many engineering structures and components due to their improved thermal, mechanical, corrosion-resistant and wear-resistant properties compared to the classical engineering materials, laminates and composites. FGMs are continuously non-homogeneous materials because the volume fractions of their composite constituents vary continuously in space. One important issue in the design, optimization and engineering applications of FGMs is concerned with their fracture and fatigue properties, which are essential to their integrity, reliability and durability [1–3]. Since the boundary value problem arising in crack analysis of FGMs is governed by partial differential equations with variable coefficients, many available analytical or numerical

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methods for homogeneous materials cannot be directly applied or easily extended to solve this class of problems. The conventional finite element method (FEM) implemented in the most commercial FEM codes assumes constant material properties within each element, which requires consequently a very fine FE-mesh for FGMs. To overcome this difficulty, graded finite elements have been proposed ([4,5]). The classical boundary element method (BEM) or boundary integral equation method (BIEM), which has been proven to be highly accurate and efficient for crack analysis in homogeneous and linear elastic materials ([6,7]), cannot be directly applied to crack analysis in FGMs either, since the required fundamental solutions or Green's functions for partial differential equations with variable coefficients can generally not be obtained in closed or simple forms. For exponentially graded materials, elastostatic Greens functions have been derived by Martin et al. [8] and Chan et al. [9].

In this paper, a BIEM is presented for elastostatic analysis of a finite crack in an infinite and linear elastic FGM subjected to an antiplane crack-face loading. An exponential law is used to describe the spatial variation of the shear modulus. Both the unidirectionally and bidirectionally graded materials are dealt with. The boundary value problem is formulated as a hypersingular traction BIE. A Galerkin method is adopted for the numerical solution of the hypersingular BIE. Fourier-integral expressions of the required Green's functions are used in the BIEM. The unknown crack-opening-displacement (COD) is approximated by a Galerkin-ansatz consisting of Chebyshev polynomials of second kind. The present BIEM requires no special regularization or integration techniques for computing the hypersingular Hadamard finite-part integral. Only a single integration is needed in spite of the application of the Galerkin method. Numerical results are presented and discussed for both unidirectional and bidirectional material gradations. The effects of the material gradients and the crack orientation on the stress intensity factors are analyzed.

The antiplane crack problem in non-homogeneous materials has been investigated previously by Erdogan [10] and Chan et al. [11], who assumed that the crack is parallel to the material gradient. In-plane cracks in FGMs have been analyzed by Delale and Erdogan [12], Konda and Erdogan [13] and Zhang et al. [14]. Transient elastodynamic analysis of an antiplane crack in a FGM has been performed by Zhang et al. [15]. However, to the authors knowledge, antiplane crack analysis in unidirectional FGMs with an arbitrary crack orientation or in general bidirectional FGMs as considered in this paper has yet not been presented elsewhere.

2. Problem statement and boundary integral equation

Consider an infinite, isotropic, continuously non-homogeneous and linear elastic FGM containing a finite crack of length $2a$ as depicted in Fig. 1. The crack is subjected to an antiplane crack-face loading. In the absence of body force, the cracked FGM satisfies the equilibrium equation

$$\sigma_{3\alpha,\alpha} = 0, \tag{1}$$

the Hooke's law

$$\sigma_{3\alpha} = \mu(\mathbf{x}) u_{3,\alpha}, \tag{2}$$

and the boundary condition on the crack-faces

$$f_3(x_1, x_2 = 0) = \sigma^*_{32}(x_1), \quad x_1 \in [-a, +a]. \tag{3}$$

In Eqs. (1)–(3), u_3 denotes the displacement component in the x_3-direction, $\sigma_{3\alpha}$ represent the shear stress components, $\mu(\mathbf{x})$ is the shear modulus, f_3 is the traction component, and σ^*_{32} is the prescribed stress loading. Also, a comma after a quantity stands for partial derivatives with respect to spatial variables, and the conventional summation convention over repeated indices is used.

The spatial variation of the shear modulus is described by an exponential law of the form

$$\mu(\mathbf{x}) = \mu_0 e^{\alpha x_1 + \beta x_2}, \tag{4}$$

where μ_0, α and β are gradient parameters of the FGM. The exponential law (4) is suitable for both unidirectional and bidirectional material gradations as shown in Fig. 1. By taking one of the gradient parameters equals zero, a unidirectional