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Lattice hydrodynamic model with bidirectional pedestrian flow

Huan-huan Tian [a], Hong-di He [b], Yan-fang Wei [c], Xue Yu [a,c,*], Wei-zhen Lu [b]

[a] Institute of Physical Science and Engineering, Guangxi University, Nanning 53004, China
[b] Department of Building and Construction, City University of Hong Kong, Hong Kong
[c] Shanghai Institute of Applied Mathematics and Mechanics, Shanghai University, Shanghai 200072, China

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ABSTRACT

The two-dimensional lattice hydrodynamic model of traffic is extended to the two-dimensional bidirectional pedestrian flow via taking four types of pedestrians into account. The stability condition and the mKdV equation to describe the density wave of pedestrian congestion are obtained by linear stability and nonlinear analysis, respectively. In addition, there exist three phase transitions among the freely moving phase, the coexisting phase and the uniformly congested phase in the phase diagram. It can also be found that the critical point a_c refers to not only the fraction c_1 of the eastbound and westbound pedestrians, but also the fraction c_2 of the northbound and southbound pedestrians. However, the critical point a_c could not appear in the phase diagram and congested crowd at any time when two fractions are equal to same value of 0.5 ($c_1 = c_2 = 0.5$). Furthermore, numerical simulation is carried out to examine the performance of such a model and the results show coincidence with the theory analysis results.

1. Introduction

Pedestrian flow dynamics is a kind of many-body system consisting of interacting individuals. In the procedure of its evolution, it shows many interesting physical phenomena such as self-organization phenomena, noise-induced ordering, and collective phenomena in panic situations such as "freezing by heating" [1], the "faster-is-slow effect" [2], and herding behavior [2]. Moreover, pedestrian movement is an important factor in the analysis and design of channels, traffic intersections, bridges, markets, and other public buildings [3]. It is necessary to know the characteristics, especially the pedestrian flow rate for rush hour and panic escape in order to avoid the unexpected accident. Therefore, pedestrian dynamics have been attracted considerable attention of scientists and engineers because of the observed non-equilibrium phase transitions, various nonlinear dynamical phenomena and safe design of public buildings. Many pedestrian traffic model have been proposed, these include the hydrodynamic models [4,5], the social force model [2,6], micro-simulation models [7], cellular automaton models [8,9], lattice gas model [10,11], emergency and evacuation models [12], and artificial-intelligence-based models [13].

Kerner and Konhaüser [14] have found the single-pulse density wave in the numerical simulation of the hydrodynamic traffic model. Kurtze and Hong [15] have proved that the single-pulse density wave is a soliton. Komatsu and Sasa [16] have derived the modified KdV equation from the optimal velocity model to describe traffic jams in terms of a kink density wave. Muramatsu and Nagatani [17] have shown that the solitonary density wave appears only near the neutral stability line. The lattice hydrodynamic model first proposed by Nagatani [18,19] incorporates the idea of the microscopic optimal velocity model, and is a simplified version of the macroscopic hydrodynamic model. Nagatani has extended the one-dimensional

* Corresponding author at: Institute of Physical Science and Engineering, Guangxi University, Nanning 53004, China.
E-mail address: yuxuegxu@gxu.edu.cn (X. Yu).

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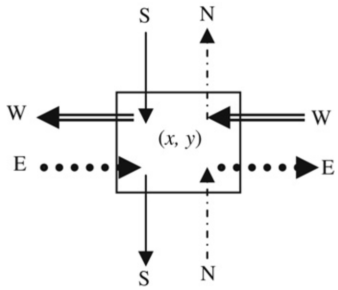

Fig. 1. The schematic diagram of the moving direction of the pedestrians. E, W, N and S denote the eastbound, westbound, northbound and southbound pedestrians, respectively.

lattice hydrodynamic model to the two-dimensional one to investigate the two-way traffic like BML model [20]. Similarly, he has also derived the mKdV equation to describe traffic jamming in the form of the kink–antikink density wave from the hydrodynamic model.

In this paper, we address whether there exist the kink–antikink density waves in pedestrian traffic. The extension of the two-dimensional lattice hydrodynamic model to the two-dimensional bidirectional pedestrian flow is taken into account by introducing the four types of pedestrians. In Section 2, the two-dimensional bidirectional pedestrian flow is proposed. And the characteristics of the two-dimensional pedestrian traffic model are analyzed by using the linear stability theory, nonlinear analysis and verifying by computer simulation in Sections 3, 4 and 5, respectively. Finally, some conclusions are given.

2. Model

The two-dimensional lattice hydrodynamic model of traffic is extended to the two-dimensional bidirectional pedestrian flow in Fig. 1 (i.e. four-way traffic). Four types of pedestrians are considered: one type of pedestrian (eastbound pedestrian) moves only to the positive x direction, one type of pedestrian (westbound pedestrian) moves only to the negative x direction, one type of pedestrian (northbound pedestrian) moves only to the positive y direction, and the other type of pedestrian (southbound pedestrian) moves only to the negative y direction.

The fraction of eastbound and westbound pedestrians in all of the pedestrians is c, the fraction of eastbound pedestrian in eastbound and westbound pedestrians is c_1, and the fraction of northbound pedestrian in northbound and southbound pedestrians is c_2. The total average density is ρ_0.

The density of eastbound, westbound, northbound, and southbound pedestrians at site (x, y) at time t are denoted by $\rho_{x+}(x, y, t)$, $\rho_{x-}(x, y, t)$, $\rho_{y+}(x, y, t)$, and $\rho_{y-}(x, y, t)$, respectively. The flux of eastbound, westbound, northbound, and southbound pedestrians at site (x, y) at time t are denoted by and $Q_{x+}(x, y, t)$, $Q_{x-}(x, y, t)$, $Q_{y+}(x, y, t)$ and $Q_{y-}(x, y, t)$, respectively. The continuity equation of the eastbound pedestrian is given by

$$\partial_t \rho_{x+}(x, y, t) + \partial_x Q_{x+}(x, y, t) = 0, \tag{1a}$$

$$\partial_t \rho_{x-}(x, y, t) + \partial_x Q_{x-}(x, y, t) = 0, \tag{1b}$$

$$\partial_t \rho_{y+}(x, y, t) + \partial_y Q_{y+}(x, y, t) = 0, \tag{1c}$$

$$\partial_t \rho_{y-}(x, y, t) + \partial_y Q_{y-}(x, y, t) = 0, \tag{1d}$$

where $\partial_t = \partial/\partial t$, $\partial_x = \partial/\partial x$, and $\partial_y = \partial/\partial y$.

At first, the case of eastbound pedestrian is considered. The lattice model of Eq. (1a) is given by

$$\partial_t \rho_{x+}(j, m, t) + \rho_{0x+}[Q_{x+}(j, m, t) - Q_{x+}(j - 1, m, t)] = 0, \tag{2}$$

where $\rho_{0x+} = c_1 \rho_{0x} = cc_1 \rho_0$. Thus, one obtains

$$\partial_t \rho_{x+}(j, m, t) + (cc_1)\rho_0[Q_{x+}(j, m, t) - Q_{x+}(j - 1, m, t)] = 0. \tag{3}$$

In terms of Nagatani's idea, the flux for the eastbound pedestrian is determined by the total optimal current with delay time.

$$Q_{x+}(j, m, t + \tau) = cc_1 \rho_0 V(\rho(j + 1, m, t)), \tag{4}$$

where the function $V(\rho(j, m, t))$ is called the optimal velocity. Eq. (4) means that the flux $Q_{x+}(j, m, t + \tau)$ of the eastbound pedestrian at site (j, m) at time $t + \tau$ are adjusted by the total optimal current $\rho_0 V(\rho(j + 1, m, t))$ at site $(j + 1, m)$ at time t. Nakayama et al. [21,22] have first introduced the optimal velocity function as Bando's model to study the pedestrian flow. Similarly, we attempt to extend the optimal velocity functions of the vehicle traffic to pedestrian traffic in order to