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Vehicular motion on a selected path in a 2d traffic network controlled by signals

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ABSTRACT

We study the dynamic behavior of vehicular traffic through a series of traffic lights on selected paths in a two-dimensional (2d) traffic network. The city traffic network is made of one-way perpendicular streets arranged in a square lattice with traffic signals where vertical streets are oriented upwards and horizontal streets are oriented rightwards. A vehicle moves through the series of signals on a path selected by the driver. The selected path is one of the straight, zigzag, and random paths in a 2d traffic network. The vehicular motion on a selected path is presented by the nonlinear-map model. Vehicular traffic exhibits very complex behavior with varying selected paths, cycle times, and vehicular density. The dependence of the arrival time on cycle time, selected path, and density is clarified for 2d city traffic.

1. Introduction

Recently, transportation problems have attracted much attention in the fields of physics [1–5]. Physics, other sciences and technologies meet at the frontier area of interdisciplinary research. The concepts and techniques of physics are being applied to such complex systems as transportation systems. The traffic flow, pedestrian flow, and bus-route problem have been studied from a point of view of statistical mechanics and nonlinear dynamics [6–27]. The jams, chaos, and pattern formation are typical signatures of the complex behavior of transportation.

Mobility is nowadays one of the most significant ingredients of a modern society. In urban traffic, vehicles are controlled by traffic signals to give priority for a road because they encounter crossings. In real traffic, vehicular traffic depends highly on the control of traffic signals. Optimizing traffic lights for city traffic has been studied by using the CA traffic model and the optimal velocity model [28,29]. The effect of signal control strategy on vehicular traffic has been clarified. It has been shown that city traffic controlled by traffic signals can be reduced to a simpler problem of a single-lane highway with a few signals. One has studied vehicular traffic controlled by a few traffic signals. It has been concluded that periodic traffic does not depend on the number of traffic lights [28,29]. Also, Lammer and Helbing have studied the effect of self-controlled signals on vehicular traffic [30,31].

Very recently, a few works have been done for the traffic of vehicles moving through an infinite series of traffic lights with the same interval. The effect of cycle time on vehicular traffic has been clarified [32–36].

Generally, traffic lights are controlled by either synchronized or green-wave strategies. In the synchronized strategy, all the signals change simultaneously and periodically where the phase shift has the same value for all signals. In the green-wave strategy, the signal changes with a certain time delay between the signal phases of two successive intersections. The

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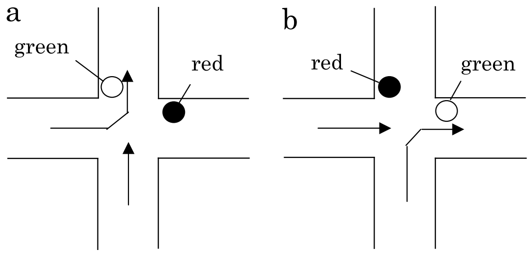

Fig. 1. Schematic illustration of signal control on a crossing. All signals on the horizontal (vertical) streets change simultaneously from red (green) to green (red). Two signals on a single crossing change from Fig. 1(a), through (b), to (a) periodically.

change of traffic lights propagates backwards like a green wave. Thus, vehicular traffic was controlled by varying the phase shift of signals. More generally, the traffic signal can be controlled by means of the phase shift (offset time), cycle time, and split time.

Researchers have investigated traffic flow through a series of signals on a single roadway. For urban traffic, vehicles do not move on a single roadway but go on a two-dimensional (2d) traffic network controlled by signals. Generally, when a driver goes from the origin to the destination, he is able to select various paths in the 2d city traffic network. Then, the arrival time depends on the selected path. However, it has been little known how the vehicular motion depends on the selected path in 2d city traffic network. The dynamical model has been little known for studying vehicular motion on a 2d city traffic network controlled by signals.

In this paper, we study vehicular traffic through a series of signals on the selected path in a 2d city traffic network. We present a nonlinear dynamic model for vehicular motion on the selected path. We investigate the dependence of vehicular motion on the selected paths: the straight, zigzag, and random paths. We clarify the vehicular dynamics on selected paths by varying the cycle time of signals and vehicular density.

2. Model and nonlinear map

We consider the motion of vehicles going through a series of traffic lights on the selected path in a 2d city traffic network. Vehicles are allowed to pass other vehicles freely. With freely passing traffic, vehicles are not correlated with each other. Therefore, we study the motion of a single vehicle in the 2d city traffic network. The city traffic network is made of one-way perpendicular streets arranged on a square lattice. Vertical streets are oriented upwards and horizontal streets are oriented rightwards. Vehicles move to North on the vertical streets and to East on the horizontal streets. The interval between the streets is l. Vehicles move with a mean speed v if there are no signals.

Traffic lights are positioned at all crossings. Fig. 1 shows the schematic illustration of signal controls at a crossing. We consider the synchronized strategy for the signal control. All east (north) signals on the horizontal (vertical) streets change simultaneously from red (green) to green (red). The signal changes periodically with period t_s. Period t_s is called the cycle time of the signal. East and north signals change alternately. If the east signals are red (green) on the horizontal streets, the north signals are green (red) on the vertical streets. Thus, two signals on a single crossing change from Fig. 1(a), through (b), to (a) periodically.

If a driver on the horizontal street wishes to turn to north and the north signal is green in Fig. 1(a), he can go north. If he wishes to turn to the north and the north signal is red, he stops at the crossing. When the north signal changes from red to green, he can go north. If the driver on the horizontal street wishes to go east and the east signal is green in Fig. 1(b), he can go straight on (east). If he wishes to go straight on (east) and the east signal is red, he stops at the crossing. When the east signal changes from red to green, he goes straight on (east).

Generally, one selects various paths on the 2d city traffic network. Here, we consider three selected paths on the square lattice: (a) straight, (b) zigzag, and (c) random paths. Fig. 2 shows the selected paths on the square lattice. The random path shown in Fig. 2(c) is a typical one. Point O indicates the origin and point D the destination. The selected paths have the same origin. The destination is different but the length (Manhattan distance) of the selected path is the same. The traffic signals on the selected path are numbered, from upstream to downstream, by $1, 2, 3, \ldots, n, n+1, \ldots$.

When a vehicle on the selected path arrives at a crossing and the signal is red, the vehicle stops at the position of the crossing. Then, when the traffic signal changes from red to green, the vehicle goes ahead. On the other hand, when a vehicle on the selected path arrives at a crossing and the signal is green, the vehicle does not stop and goes ahead without changing speed.

We define the arrival time of the vehicle at traffic light n as $t(n)$. The phase shift t_{phase} of the north signal is different from that of the east signal by a half cycle time $t_s/2$. Then, the arrival time at traffic light $n+1$ is given by

$$t(n+1) = t(n) + l/v + (r(n) - t(n))\, H\left(t(n) + t_{phase}(n) - \left[\left(\mathrm{int}((t(n) + t_{phase}(n))/t_s)\right) t_s\right] - t_s/2\right)$$
$$\text{with } r(n) = \left(\mathrm{int}\left((t(n) + t_{phase}(n))/t_s\right) + 1\right) \cdot t_s - t_{phase}(n), \quad (1)$$