Physica A 388 (2009) 2931–2940

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Choosing effective controlled nodes for scale-free network synchronization

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ARTICLE INFO

Article history:
Received 18 December 2008
Received in revised form 18 March 2009
Available online 31 March 2009

PACS:
89.75.Hc
05.45.Xt
05.45.Gg

Keywords:
Pinning control
Synchronization
Unweighted symmetrical scale-free network
Normalized weighted scale-free network

ABSTRACT

Previous studies concerning pinning control of complex-network synchronization have very often demonstrated that in an unweighted symmetrical scale-free network, controlling the high-degree nodes is more efficient than controlling randomly chosen ones; due to the heterogeneity of the node-degree or edge-connection distribution of the scale-free network, small-degree nodes have relatively high probabilities of being chosen at random but their control has less influence on the other nodes through the network. This raises the question of whether or not controlling the high-degree nodes is always better than controlling the small ones in scale-free networks. Our answer to this is *yes* and *no*. In this study, we carry out extensive numerical simulations to show that in an unweighted symmetrical Barabasi–Albert scale-free network, when the portion of controlled nodes is relatively large, controlling the small nodes becomes better than controlling the big nodes and controlling randomly chosen nodes has approximately the same effect as controlling the big ones. However, we also show that for normalized weighted scale-free networks, controlling the big nodes is in fact always better than controlling the small ones.

1. Introduction

Pinning control is an efficient method for dealing with complex networks; it can drive the network to a desired state by controlling just a small portion of nodes. Pinning control is especially effective for achieving network synchronization, among other control objectives, and therefore has been applied frequently in recent years [1–16].

Two main problems encountered in the studies of pinning-controlled synchronization are those of how to choose the controlled nodes and how to design the controllers. There have many studies (e.g., Refs. [1–4]) focused on the second problem, and this paper focuses on the first issue. It is clear that the choice of controlled nodes should depend on the network structure. In a homogeneous network, such as a fully connected network, a nearest-neighbor coupling network, or, particularly, a random network, every node has roughly the same number of links, so every node in the network has about the same importance. Thus, which nodes are chosen for controlling has little effect on the control performance. However, in a heterogeneous network, typically a scale-free network, the node degrees obey a power-law distribution and therefore most nodes have small degrees (these are called small nodes, and have small numbers of connections) and only a few nodes have high degrees (these are called big nodes, and have large numbers of connections); each node in the network has very different importance. Obviously, in the latter case, which nodes are chosen for controlling has a significant impact on the control results.

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doi:10.1016/j.physa.2009.03.040

Previous studies (e.g., Refs. [5–7]) have very often shown that in an unweighted symmetrical scale-free network, given a pre-assigned number of controlled nodes, controlling the big nodes is more efficient than controlling randomly chosen ones. The main reason given is that due to the heterogeneity of the degree distribution of such networks, small nodes have relatively high probabilities of being chosen at random, but they have little influence on the other nodes through the network. This raises the natural question of whether or not controlling the big nodes is always better than controlling the small ones in a scale-free network. Our answer is *yes* and *no*. In this study, we will show that in an unweighted symmetrical Barabasi–Albert (BA) scale-free network [17], when the pre-assigned portion of controlled nodes is relatively large, controlling small nodes turns out to be better than controlling the big nodes and controlling randomly chosen nodes has approximately the same effect as controlling the big ones. However, we also show that for normalized weighted scale-free networks, controlling the big nodes is in fact always better than controlling the small ones.

More precisely, the study in Ref. [7] has shown that the analysis of pinning-controlled synchronization of a network with N nodes can be converted to the analysis of uncontrolled synchronization of its extended network with $N+1$ nodes, so the master stability function (MSF) method [10] can be applied to analyze the original controlled synchronizability. On the other hand, the studies in Ref. [12] have shown that the synchronized regions of various complex networks can be classified into two cases: bounded and unbounded, and more recent studies [8,9,18] show that they can also be a certain combination of these. For a symmetric network with a real eigenspectrum, when the synchronized region is unbounded the network synchronizability is determined by the smallest nonzero eigenvalue of the coupling matrix, while if the region is bounded then it is determined by the eigenratio of the largest and the smallest nonzero eigenvalues. For a weighted asymmetrical network with a complex eigenspectrum, they are changed to being the real part of the smallest nonzero eigenvalue and the eigenratio of the real parts of the largest and the smallest nonzero eigenvalues, respectively [11,14,16]. So, to generalize these results to complex networks in more general forms, the real parts of the eigenvalues of the coupling matrices are considered here for the two aforementioned (bounded and unbounded) cases of network synchronization regions.

This study compares three representative node-picking schemes. (i) Scheme 1: arranging all nodes according to their descending degrees; then, controlling the nodes in descending order according to their degrees. (ii) Scheme 2: arranging all nodes according to their ascending degrees; then, controlling the nodes in ascending order according to their degrees. (iii) Scheme 3: controlling nodes chosen at random. These three schemes are applied to two kinds of scale-free networks, namely, unweighted symmetrical networks and normalized weighted asymmetrical networks, with comparison through extensive numerical simulations.

In the rest of this paper, firstly the problem of pinning-controlled synchronization of a network with N nodes is converted to the problem of uncontrolled synchronization of its extended network with $N+1$ nodes. Then, the effects of the feedback control gain and the portion of controlled nodes on the controlled synchronizability are studied, through extensive numerical simulations, where the synchronizability is described according to the two cases of bounded and unbounded regions via the real part of the smallest nonzero eigenvalue and the eigenratio, respectively. It is shown that in an unweighted symmetrical BA scale-free network, when the portion of controlled nodes is relatively large, Scheme 2 is better than Scheme 1 and Scheme 3 is similar to Scheme 1 with a relatively large average number of edges. It is also shown that in the limiting case where the control gain approaches infinity and the portion of controlled nodes approaches 1 (=100%), Scheme 2 is the best one among the three, while Scheme 3 is better than Scheme 1. For a normalized weighted asymmetrical scale-free network, however, it is shown that Scheme 1 is the best one among the three, while Scheme 3 is better than Scheme 2 except for the limiting case where the control gain approaches infinity and the portion of controlled nodes approaches 1.

2. The pinning-controlled network model

Consider the following network model:

$$\dot{\mathbf{x}}_i = \mathbf{F}(\mathbf{x}_i) - \sigma \sum_{j=1}^{N} G_{ij}\mathbf{H}(\mathbf{x_j}), \quad i = 1, \ldots, N, \tag{1}$$

where $\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x})$ denotes the dynamics of each individual node, $\mathbf{H}(\mathbf{x})$ is a vector-valued inner-coupling function, σ is the overall coupling strength, $[G_{ij}]$ is a zero-row-sum coupling matrix which has information about the topology and weights of the network.

With pinning controllers, the dynamics of the controlled network is described by

$$\dot{\mathbf{x}}_i = \mathbf{F}(\mathbf{x}_i) - \sigma \sum_{j=1}^{N} G_{ij}\mathbf{H}(\mathbf{x}_j) + \sigma\delta_i u_i, \quad i = 1, 2, \ldots, N, \tag{2}$$

where the last item represents the pinning controllers, which exist only at the controlled nodes in the index set $C = \{i_1, i_2, \ldots, i_n\}$, in which $n = \lfloor pN \rfloor$ for a parameter $p << 1$ and $\lfloor a \rfloor$ is the integer part of real number a: if $i \in C$, then $\delta_i = 1$; otherwise, $\delta_i = 0$.

Suppose that $\mathbf{s}(t)$ is a target state satisfying $\dot{\mathbf{s}}(t) = \mathbf{F}(\mathbf{s}(t))$, and this state information is available for feedback. Then, by applying a simple error-feedback controller to each controlled node, i.e., with $u_i = k_i(\mathbf{H}(s) - \mathbf{H}(x_i))$ at controlled node i,