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ScienceDirect
Physica A 387 (2008) 1446–1454
PHYSICA A
www.elsevier.com/locate/physa

Stochastic resonance of a periodically driven neuron under non-Gaussian noise

J.R.R. Duarte, M.V.D. Vermelho, M.L. Lyra*

Instituto de Física, Universidade Federal de Alagoas, 57072-970 Maceió - AL, Brazil

Received 29 August 2007; received in revised form 25 October 2007
Available online 13 November 2007

Abstract

We investigate the first-passage-time statistics of the integrate–fire neuron model driven by a sub-threshold harmonic signal superposed with a non-Gaussian noise. Here, we considered the noise as the result of a random multiplicative process displaced from the origin by an additive term. Such a mechanism generates a power-law distributed noise whose characteristic decay exponent can be finely tuned. We performed numerical simulations to analyze the influence of the noise non-Gaussian character on the stochastic resonance condition. We found that when the noise deviates from Gaussian statistics, the resonance condition occurs at weaker noise intensities, achieving a minimum at a finite value of the distribution function decay exponent. We discuss the possible relevance of this feature to the efficiency of the firing dynamics of biological neurons, as the present result indicates that neurons would require a lower noise level to detect a sub-threshold signal when its statistics departs from Gaussian.
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PACS: 05.40.-a; 02.50.Ey; 87.19.La

Keywords: Stochastic resonance; Non-Gaussian noise; Integrate–fire model

1. Introduction

The idea of stochastic resonance has been widely applied in investigating many physical, chemical and biological systems, including optical, electronic and magnetic systems [1], chemical reactions [2] and neuro-physiological aspects of sensory systems [3–5]. It has been extensively investigated in dynamical models of periodically stimulated sensory neurons [1,6–8]. In the basic integrate–fire neuron model the state of the neuron is described in terms of its membrane potential resulting from synaptic inputs. When the membrane potential reaches a threshold, a spike is generated indicating an action potential. The spike train exhibits a statistical phase lock to the sub-threshold stimulus added to noise. The distribution of the interspike intervals, like the first-passage-time distribution, presents regular peaks signaling the sub-threshold signal. The intensity of these peaks goes through a maximum as the noise intensity is increased.

Analytical and numerical studies of stochastic resonance usually consider the noise to be uncorrelated in time (white) and Gaussian, a good approximation to model systems where the relaxation time of the noise auto-correlation

* Corresponding author.
E-mail address: marcelo@fis.ufal.br (M.L. Lyra).

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doi:10.1016/j.physa.2007.11.011

is much shorter than the characteristic time scale of the dynamical system. The effect of the noise correlation time in a bistable system was first investigated by Gammaitoni et al. [9] showing a degradation of the resonance effect due to the competition between the noise correlation time and the average time of waiting between noise-induced inter-well transitions. Correlations also play a relevant role in the stochastic resonance of neuron models [10–13]. An experimental study of the effect of correlated (colored) noise in the stochastic resonance of sensory neurons showed that, for low frequencies of the periodic signal, conventional white noise presents the lowest optimal noise intensity and the highest signal-to-noise ratio as compared with colored noise [14]. However, the same study suggested that colored $1/f$ noise may be better than white noise at high frequencies, providing a possible explanation for the wide occurrence of $1/f$ noise in biological systems, a feature reinforced by more recent works [15,16].

Stochastic resonance induced by colored and non-Gaussian noises has also been recently investigated [17–21] showing an enhancement on the signal-to-noise ratio when the noise departs from Gaussian behavior. Also, a numerical investigation of stochastic resonance in bistable systems driven by a white noise with power-law distributed intensities showed that an optimal transition rate can be achieved for a finite decay exponent of the noise probability distribution [22]. There is a growing interest in studying dynamical systems driven by non-Gaussian noises with slowly decaying power-law distribution, given that they are quite ubiquitous in natural phenomena [23]. One of the simplest mechanisms for generating a power-law distributed noise is through a random multiplicative process (RMP) [24–26]. This mechanism has been widely used to model stochastic series emerging, for example, in economics [27–29] and biology [30–32]. It has been shown that, when the multiplicative random process acts together with an additive noise term, or more generally when the dynamical variable is repelled from the origin, true power-law distributed random series can be generated [24,25].

In this work, we will study the dynamics of the integrate and fire model for the neural response driven by a periodic sub-threshold signal and under the influence of a non-Gaussian noise generated by a random multiplicative process. We will be particularly interested in evaluating the first-passage-time distribution whose peaks may reveal the main time scale of the underlying periodic signal. The intensity of these peaks passes through a maximum when varying the noise intensity, a typical signature of stochastic resonance. We will give a detailed analysis of the resonance condition as a function of the noise statistics to show that optimal efficiency in recognizing the sub-threshold signal can be achieved at weaker noises when its statistics departs from the Gaussian behavior.

2. Model and numerical procedure

The integrate–fire neuron model has been widely used as the standard model for investigating the dynamics of neural systems. It is able to qualitatively describe the sub-threshold integration which occurs on a time scale much slower than that involved in the spike generation. Within this approach, the membrane potential of a periodically driven neuron is assumed to obey the stochastic differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\gamma x + \mu + A\sin(\omega t + \phi) + v(t), \tag{1}$$

where γ is the inverse of the membrane time constant and μ/γ is the equilibrium membrane potential in the absence of external inputs. $v(t)$ represents a source of noise for the synaptic inputs which superposes with the periodic stimulus. The above stochastic equation is to be complemented with a spike-and-reset rule. Whenever the membrane voltage x reaches a threshold value Θ, a spike is generated and the membrane potential is reset to $x = 0$.

In order to investigate the stochastic resonance, the amplitude of the periodic input should be weak, so that the oscillations that it induces in the membrane voltage are not enough to promote the threshold crossing. The threshold crossing and the consequent neuron spike are ultimately due to the presence of noise. In what follows, we will be particularly interested in the statistics of the first-passage time. This quantity can be obtained analytically from simplified approaches [34–37], thus allowing a direct verification of numerical results. Other relevant quantities are related to the spike sequence and the interspike-interval distribution. For an endogenous periodic input, which is reset to its initial phase after each spike (taken to be $\phi = 0$), these two distributions coincide. Endogenous stimuli are biologically unrealistic and some features are artifacts of such reduced dynamics [38]. For exogenous stimuli, the statistics of the first-passage time and the interspike intervals are distinct, especially when neuron spikes are induced at weak noise. However, noise-induced phenomena found in the interspike-interval distribution show comparable resonance effects with and without the stimulus reset [34,39,40]. The average interval between spikes, like the average

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