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Dynamics of traffic networks: From microscopic and macroscopic perspectives

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Abstract

Considering the microscopic characteristics (vehicle speed, road length etc.) of links and macroscopic behaviors of traffic systems, we derive the critical flow generation rate in scale-free networks. And the dynamics of traffic congestion is studied numerically in this paper. It is shown that the queue length increases with microscopic characteristics of links. Additionally, the critical flow generation rate decreases with increase of the network size N, maximum speed $v_{\max}$ and parameter τ. The significance of this finding is that, in order to improve the traffic environment, both the local information for the single link and behaviors of the whole network must be analyzed simultaneously in a traffic system design.

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1. Introduction

The structure and dynamics of complex networks have attracted a tremendous amount of recent interest [1–3] since the seminal works on scale-free networks by Barabási and Albert [4] and on the small-world phenomenon by Watts and Strogatz [5]. Mathematically, a way to characterize a complex network is to examine the degree distribution $P(k)$, where k is used to measure the number of links at a node. Scale-free networks are characterized by $P(k) \sim k^{-\lambda}$, where k is the algebraic scaling exponent [4].

Recently, more and more researchers have begun to develop models for explaining the dynamic behaviors of traffic on complex networks, i.e. the critical value of the flow generation rate and the cascade failure [6–8]. A detailed analysis of dynamic behaviors caused by traffic congestion in gradient networks suggests that there exists a critical value of the average degree. For values of the average degree below this critical value, large scale-free networks are somewhat more prone to congestion than random networks with the same number of nodes and average degree while the opposite is true above the critical value [9]. In particular, Zhao et al. [6] propose a cascade failure model for complex networks and

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uncovers a congestion phase-transition phenomenon in terms of the key parameter characterizing the node capacity. Wu et al. [10] derive the dynamic critical flow generation rate based on the unoccupied capacity in scale-free networks.

A common feature in previous studies is that they don't consider the microcosmic characteristics of traffic problem such as road length, vehicle speed, density etc. Of course, there are many studies on microcosmic traffic flow models [11–13], in which traffic phenomena on a single link are analyzed, while traffic behaviors on the whole network are not considered. The quantity of interest is the critical rate R_c of flow generation (as measured by the number of flows created within the network in unit time), at which a phase transition occurs from free to congested traffic flow. For $R < R_c$, the numbers of created and delivered flows are balanced, resulting in a steady state, or free flow of traffic. The network is in an uncongested state, while for $R \geq R_c$, congestions occur in the sense that the number of accumulated flows increases with time, due to the fact that the capacities of links for delivering flows are limited. We are interested in determining the phase-transition point R_c, given a network topology. Inspired by previous work [11], we study both microscopic behaviors of individual vehicles on links and the statistical properties of traffic networks. In this paper, the main point is deriving the critical flow generation rate of the traffic network based on the microscopic characteristics. Further we study how the statistical properties, both at microscopic (road length, vehicle speed, etc.) and macroscopic levels (behaviors on the whole network), vary with the flow generation rate R.

2. Model of critical flow generation rate

In this section, we derive the critical flow generation rate by recalling the model proposed by Mahnke and Pieret [11] first. For complete details, we refer the reader to the reference. For succinctness, we present the notation used for this model as follows.

S: the mean size of the traffic congestion or the queue length of traffic congestion; F: the total number of vehicles; L: the total length of the road; l_0: the effective length of a single vehicle; τ: the average time of the head vehicle changing its state from congestion to free; d: a positive control parameter, which can be seen as a characteristic headway for the transition between noninteracting and interacting phases; $v_{\max}$: the maximal speed allowed.

In Ref. [11], a stochastic description of congestion formation using the master-equation approach is given, and the transition probabilities for the jump processes are constructed. The following equation holds:

$$S = F - \frac{L/l_0 - F}{(v_{\max}\tau/2l_0 \pm \sqrt{(v_{\max}\tau/2l_0)^2 - (d/l_0)^2})}. \tag{1}$$

The queue is formed when $S \geq 0$ and the traffic congestion occurs. Therefore, the critical flow generation rate R_c can be obtained when $S = 0$.

To account for the network topology, we assume that the capacities for processing flows P are different for different links, depending on the degree of nodes connecting this link. Then, $P_{ij} = \beta(k_i + k_j)$, where P_{ij} is the processing capacity of link $i - j$, k_i is the degree of node i and β is the parameter.

In given manual networks, starting from an unloaded network, the evolving mechanism of traffic flow can be described as:

(1) Generation traffic flow. Assumed that the flows are generated at the nodes. At each time step, we impose a constant input of newly created traffic flow R. The source of each flow as well as its destination is chosen at random among all the nodes of the network. Besides, each node can send P flow which is related to the degree of the node at each time step and, as a consequence, one node, i, can have a queue to be delivered.
(2) Movement. Flows move through the graph simultaneously searching for their respective destinations based on the shortest path algorithm. At each time step, only at most P_{ij} flows can be transported on link $i - j$ according to the FIFO (First-In–First-Out) principle. When the queue at a selected link is full, the link won't accept any more flows and the flow will wait for the next opportunity. Once a vehicle arrives at its destination, it will be removed from the system [14].

Because the link with the largest betweenness can be easily congested and the congestion can quickly spread to the entire network, it is necessary to consider only the traffic balance of this link. Since the flows are transmitted along the shortest paths from the source to the destination, the probability that a created flow will pass through the link with the largest betweenness i is $B_i/\sum_j B_j$ [6]. At each time step, on average, R flows are generated. Thus, the average