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Relativistic transformation of the canonical distribution function in relativistic statistical mechanics

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Abstract

We deduce a relativistic transformation of the canonical distribution function from basic principles; that is, from relativistic canonical transformations of dynamical variables. The thermodynamics which is obtained from such a distribution coincides with the recent proposal put forward by Ares de Parga et al. [G. Ares de Parga, B. López-Carrera, F. Angulo-Brown, J. Phys. A 38 (2005) 2821] of a relativistic renormalized thermodynamics. The invariances of the canonical distribution and the partition function are obtained. By noticing a mistake committed by Balescu while dealing with the partition function of an ideal gas, the consistency of the theory is showed. The Maxwell distribution function of an ideal gas moving with a constant velocity with respect to a reference frame is calculated. A natural asymmetry appears as a difference with the distribution function at the same temperature but at rest with respect to the same reference frame. The quantum case and the covariance of the theory are analyzed.

Keywords: Relativity; Transformation; Invariance; Thermodynamics

1. Introduction

Nowadays, the study of irreversible relativistic thermodynamics is necessary for understanding some aspects of astrophysics and cosmology. Consequently, the interest in knowing the relativistic transformation laws of the thermodynamic quantities has been revived. Nevertheless, Sieniutycz [1] still remarks that some authors conclude that equilibrium statistical mechanics cannot provide an unambiguous answer to the relativistic transformation formulae of thermodynamic quantities. Indeed, in the sixties Curie [2], Ott [3], Balescu and Kotera [4], Piña [5], Arzeliès [6] and van Kampen [7], among others, developed the basis of a covariant formulation of relativistic statistical mechanics. The effort has been achieved by the final formulation realized by Balescu [8] who explained that among the different proposals for the relativistic transformation laws of the thermodynamic quantities, the Einstein and Planck theory was the unique model which leaves invariant the form of thermodynamics. However, the theory presents the problem of not being invariant under a change of reference frame as Ares de Parga et al. [9] have proved. Indeed, some authors [10] considered the impossibility of a universal relativistic temperature transformation. Recently, Ares de

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Parga et al. [9] have demonstrated that in order to maintain the invariant form of relativistic thermodynamics, it is necessary to redefine the concept of energy. As a consequence of this, it can be shown that some ambiguities of the Balescu relativistic statistical thermodynamics [8] may be explained by applying the simplicity of the new approach. In particular, the renormalized or thermal energy will be applied to play an important role in order to understand the meaning of the partition function. Indeed, as an example of the applicability of his relativistic statistical theory, Balescu [8] committed a mistake showing the invariance of the partition function for the ideal gas. The error consists in performing a change of variables of 3 components by using the result for 4 components. This main point permits us to clarify the relation between the relativistic renormalized thermodynamics [RRT] [9] and the relativistic statistical theory presented in this paper.

The paper is organized as follows. Section 2 presents a summary of the RRT and the Balescu's deduction of the distribution function. In Section 3, the relativistic transformation of the distribution function is obtained and then the RRT is deduced. By showing the mistake committed by Balescu [8], in Section 4, we analyze the invariance of the partition function and the distribution function. The Maxwell distribution function is calculated for the ideal gas case. The quantum case is analyzed by calculating the form of the relativistic transformation of the quantum partition function in Section 5. Section 6 is advocated to the discussion of the covariant form of the RRT. Some concluding remarks are given in Section 7.

2. The renormalized relativistic thermodynamics and Balescu distribution function

Let us start by presenting a summary of the RRT. First of all, the concept of energy is redefined to what they have called the renormalized or thermal energy which is obtained by a renormalization process and it consists of subtracting the bulk energy to the regular energy; that is

$$\xi = E - W_{o\to E_o}, \tag{1}$$

where ξ, E and $W_{o\to E_o}$ represent the thermal energy, the Einstein–Planck [EP] relativistic transformation of the energy [11] and the bulk energy, respectively. The bulk energy is defined as:

$$W_{o\to E_o} = \int_0^{E_o} \vec{u}\cdot \mathrm{d}\vec{G}, \tag{2}$$

where $\vec{G}$ represents the total momentum of the system with respect to the rest frame [11], and it is given by,

$$\vec{G} = \gamma(E_o + pV_o)\vec{u}, \tag{3}$$

where the speed of light $c = 1$, $\gamma = 1/\sqrt{1-u^2}$, with u the relative velocity between both the proper and the laboratory systems, p the pressure, and V_o and E_o the volume and the energy of the system in the rest frame, respectively. It has to be noticed that in Eq. (2), $\vec{u}$ must be considered as a constant because it represents the relative uniform velocity between two reference frames. Eq. (1) can be now written as

$$\xi = E - \vec{u}\cdot\vec{G} = E - \gamma(E_o + pV_o)u^2. \tag{4}$$

Once the thermal energy is defined, we are obligated to add to the work the bulk energy obtaining the thermal work; that is:

$$\mathrm{d}\Omega = \mathrm{d}W + \vec{u}.\mathrm{d}\vec{G}. \tag{5}$$

After these definitions, we are able to transform the thermal energy, the thermal work and other extensive thermodynamical functions as follows:

$$\Gamma = \gamma^{-1}\Gamma_o, \tag{6}$$

where Γ represents any extensive thermodynamical function in the laboratory frame and the subscript $_o$ indicates that the quantity is described in the rest frame. The intensive functions are invariant; that is:

$$\varsigma = \frac{\Gamma}{V} = \frac{\Gamma_o}{V_o} = \varsigma_o. \tag{7}$$