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Spin-1/2 and spin-1 Ising model with crystal field on a bilayer Bethe lattice

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Abstract

We consider a system consisting of two layers of Bethe lattices each with a branching ratio of q Ising spins. The layer with spin-1/2 atoms interacting with the nearest-neighbor (NN) bilinear interaction J_1 is laid over the top of the other with spin-1 atoms interacting with the bilinear NN interaction J_2 and the crystal field interaction Δ, and the two layers are tied together via the bilinear interaction between the vertically aligned adjacent NN spins denoted as J_3. The exact recursion relations in a pairwise approach was employed for the solution of the problem on the bilayer Bethe lattice and the emphasis was especially given to the crystal field effects in obtaining the phase diagrams of the model. After studying the ground state (GS) phase diagrams and the thermal behaviors of the order-parameters, the temperature dependent phase diagrams of the model are obtained by considering only the ferromagnetic ordering of the layers, i.e. $J_1 > 0$ and $J_2 > 0$, and the ferromagnetic or antiferromagnetic ordering of the adjacent spins of the layers, $J_3 > 0$ or $J_3 < 0$, respectively. Besides the second- and first-order phase transitions, the model also presents compensation temperatures for appropriate values of the system parameters. The paramagnetic phase is divided into two phases by studying the thermal behaviors of the quadrupolar moment for the lower layer containing only spin-1 atoms.

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1. Introduction

The molecular-based magnetic materials with spontaneous magnetic moments are of great interest for their potential applications such as in thermomagnetic recording and in devices Ref. [1]. The mixed-spin systems in comparison with one type of spin systems present less translational symmetry, thus it is believed that ferrimagnetic ordering plays a crucial role in some of these materials, therefore, the synthesis of new ferrimagnetic materials is an active field in material science. The ferrimagnetic materials consist of two unequal magnetic moments, i.e. spin-S and spin-σ with $S \neq \sigma$, which interact antiferromagnetically, therefore, their moments do not cancel each other at low temperatures except at the compensation temperatures. The existence of compensation temperatures is the manifestation of the mixed-spins, i.e. ferrimagnets, and has an interesting application such as the magneto-optical recording Ref. [2].

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In addition, the study of magnetic thin films consisting of various magnetic layered structures or superlattices has also been receiving intense attention for both theoretical and experimental reasons Ref. [3]. These materials are made up with multiple layers of different magnetic substances, thus there is a high potential for technological advances in information storage and retrieval and in the synthesis of new magnets for a variety of applications Ref. [4]. They also present some interesting novel magnetic properties such as giant magnetoresistance Ref. [5], surface magnetic anisotropy Ref. [6], enhanced surface magnetic moment Ref. [7] and surface magnetoelastic coupling Ref. [8].

Therefore, a variety of configurations of spins on a bilayer Ising model system with spin-1/2 on one layer and a higher spin on another has been attracting a great deal of attention recently. A ferromagnetic amorphous bilayer system consisting of two monolayers (A and B) with different spins ($S_A = 1/2$ and $S_B = 1/2, 1$) and different interaction constants coupled together with an interlayer coupling was studied by using the effective field theory (EFT) Ref. [9]. The critical temperature and the layer longitudinal magnetizations of a ferromagnetic or ferrimagnetic mixed Ising bilayer system with both spin-1/2 and spin-1 (or spin-3/2) in a crystal field were investigated by using EFT with a probability distribution technique Ref. [10]. The ferromagnetic or ferrimagnetic bilayer system consisting of two magnetic monolayers (A and B) with different spins ($S_A = 1/2$ and $S_B = 1, 3/2$) and different interaction constants coupled together in a transverse crystal field were studied within the framework of EFT with correlations Ref. [11]. The differential operator technique is applied in studying the ferrimagnetic bilayer system with biaxial crystal field within the framework of EFT Ref. [12]. A ferrimagnetic bilayer system consisting of spin-1/2 and spin-3/2 Ising layers in an applied transverse field were examined by the use of EFT Ref. [13]. A ferromagnetic amorphous bilayer system consisting of two magnetic monolayers with spin-1/2 and spin-3/2 and different interaction constants coupled together with interlayer coupling was studied by the use of the EFT Ref. [14]. Transition temperatures of a bilayer system with $(A)_2(A_pB_{1-p})_1(B)_2$ consisting periodically of two layers of spin-1/2 A atoms, two layers of spin-3/2 atoms and an interface with alloying type (A_pB_{1-p}) disorder were examined by using the EFT Ref. [15]. The effects of an applied transverse magnetic field on magnetic properties in a ferrimagnetic bilayer system with disordered interfaces consisting of spin-1/2 and spin-3/2 atoms were investigated with the use of the mean-field theory (MFT) Ref. [16]. A magnetic bilayer system with s.c. symmetry consisting of spin-1/2 and spin-1 Ising spins were investigated by using the cluster variational method in the pair approximation (CVMPA) Ref. [17]. The spin configurations in the absence of an external magnetic field have been systematically investigated for a magnetic bilayer system of two ferromagnetic layers separated by a non-magnetic layer with exchange interlayer coupling Ref. [18].

The exact solutions for the realistic systems on regular lattices are generally unavailable, therefore, one usually relies on approximation methods to obtain, at least, a qualitative picture for the phase diagrams of the considered system. Thus, one may also introduce a lattice-like fictitious tree to find exact or approximate solutions of the model. A Bethe lattice is such a lattice, which is an infinitely Cayley or regular tree that is a connected graph without circuits and historically gets its name from the fact that its partition function is exactly that of an Ising model on the Bethe approximation Ref. [19]. The importance is that the Bethe lattice is an infinitely tree which gives us the negligible boundary effects, therefore, far from the boundary sites that is deep inside the Cayley tree, now Bethe lattice, all the sites become equivalent, thus studying the behavior of one spin, named as the central spin, is enough to obtain the full picture of the system. We should also comment that the Bethe lattice calculations provides exact solutions and results of which qualitatively better approximations for the regular lattices than solutions obtained by the conventional mean-field theories Ref. [20]. In addition, the cluster variation method in the pair approximation studies on regular lattices yield results that are exact for the same model on the Bethe lattice Ref. [21]. The Bethe lattice considerations also have some limitations that is it predicts a transition temperature higher than that for a regular lattice and it is not reliable for predicting critical exponents Ref. [22], where also the correspondence of the Bethe lattice with regular lattices and real physical systems and whether it can be embedded into a finite-dimensional Euclidean space are also discussed.

As a result, we consider the bilayer Ising model on the Bethe lattice with one of the layers containing only spin-1/2 atoms and the other having only spin-1 atoms each of which are allowed to interact ferromagnetically with bilinear intralayer interactions J_1 and J_2, respectively, with the NN spins from their own layers. Besides, the adjacent NN spins of the two layers are also tied together via an interaction between the vertically aligned spins denoted as J_3, which is taken to be either ferromagnetic or antiferromagnetic type. Meanwhile, the crystal field for the lower layer containing the spin-1 atoms are also included into the Hamiltonian. The problem was solved in terms of the exact recursion relations in a pairwise approach on the Bethe lattice Ref. [22,23]. After obtaining the GS phase diagrams on different planes for the given system parameters and the variations of the order-parameters, the temperature dependent phase diagrams of the model are calculated by studying the variations of the order-parameters and the free energy of the system.