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Hydrodynamics of compressible liquids: Influence of the piston effect on convection and internal gravity waves

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Abstract

The onset of stability in strongly compressible liquid with fixed volume is found by linear stability analysis. The results coincide with those at fixed pressure proving thereby that the piston effect does not influence the onset of convection. However, the dynamic phenomenon of internal gravity waves essentially depends on the piston effect.

Keywords: Compressible fluids; Convection; Internal gravity waves; Piston effect

1. Introduction

Convective instability and internal gravity waves in liquids are phenomena related to the presence of a significant height density inhomogeneity. Such non-homogeneity may appear as a result of gravity or of an external temperature gradient ∇T, or of both of these. Gravity induced stratification usually occurs in large scale geophysical objects. However, similar behavior takes place also near the liquid–gas critical points where the overall change of the density may reach 10% in a layer of a few centimeters high. Our approach is equally applicable to geophysical objects and near-critical fluids, where the compressibility is essential. Consider the layer of a liquid restricted by horizontal planes $z = 0$ and L with infinite extent in the horizontal x–y plane, subjected to a vertical gradient of temperature in the gravitational field $g\hat{z}$. The two important equilibrium properties of a liquid approaching the liquid–gas critical point are the huge (theoretically infinite) increase of the compressibility, $\alpha_T = (\partial\rho/\partial p)_T$, and of the thermal expansion, $\beta = -(1/\rho)(\partial\rho/\partial T)_p$, so that near the critical point a fluid becomes highly compressible and highly expandable. Under gravity, a highly compressible fluid collapses under its own weight to form a large density gradient. Additionally, if the heat is added from below, say by applying an external temperature gradient, a strong vertical stratification of density is established. Heating the bottom more than the top causes the fluid at the top to be denser and heavier, and creates the possibility for an instability to occur. If the temperature gradient does not exceed some critical value, the liquid remains in mechanical equilibrium. However, if this condition is not satisfied, a vertical motion of the liquid (free convection) develops tending to equalize the temperature throughout the fluid. Convection is the third (additional to thermodiffusion and radiation) mechanism of heat transfer. Convection

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is hindered by the compressibility of the liquid and dissipative processes. The large stratification leads to the appearance of horizontal oscillations of the liquid (internal gravity waves). The common to convection and internal gravity waves is an existence of a significant height density inhomogeneity. Along with the stationary convection, the oscillatory branch of convective instability may appear in a non-compressible liquid [1], which, however, remains beyond our analysis. The problem of convection and internal gravity waves in near-critical fluids came to our attention a long time ago [2–5]. The necessity of considering this effect again stems from the recently proposed outstanding theoretical idea of speeding up of the critical dynamics [6], which was subsequently found experimentally [7].

The slowing-down of the dynamic processes near the liquid–gas critical points is the hallmark of the critical phenomena. This fact is just the one which makes so difficult to get reliable equilibrium experimental data near the critical points. The slowing-down arises from the vanishing of the kinetic coefficients at the critical point, such as the heat conductivity which is inversely proportional to the specific heat at constant pressure, which is strongly singular at the critical point. The usual experimental set-up is that of a closed cell filled with fluid, where, after changing the temperature at the bottom or around the cell, one waits for the establishment of thermal equilibrium inside the cell. The slow diffusive heat propagation is responsible for the critical slowing-down. It turns out, however [6], that a fast thermal equilibrium is established through the thermo-acoustic effect, where the temperature change at the surface induces acoustic waves which result in a fast change of the pressure, and hence of the temperature, everywhere in the fluid. This effect is also called the "piston effect", since the thermal boundary layer generated near the heated bottom of the cell produces, like a piston, a pressure on the fluid confined in the constant volume. In fact, the piston effect is the fourth mechanism of heat transfer, where the heat is transferred through adiabatic compression.

The aim of this article is to calculate the influence of the piston effect on the onset of convection and on the internal gravity waves. We consider these two phenomena together since both of them are described by the hydrodynamic equations which we will analyze in the next section for a layer of a compressible liquid subjected to vertical gravity and temperature fields [8].

2. Main equations

The hydrodynamic equations are nothing but the conservation laws for mass, momentum and energy, namely the continuity equation

$$\frac{\mathrm{d}\rho'}{\mathrm{d}t} + \rho' \operatorname{div} v' = 0, \tag{1}$$

the Navier–Stokes equation

$$\frac{\mathrm{d}v'}{\mathrm{d}t} = -\frac{1}{\overline{\rho}} \nabla p' - \frac{g\rho'}{\overline{\rho}} \hat{z} + v\nabla^2 v' + \left(\frac{v}{3} + \zeta\right) \nabla \operatorname{div} v', \tag{2}$$

and the heat conductivity equation

$$\frac{\mathrm{d}Q}{\mathrm{d}t} \equiv \rho T' \frac{\mathrm{d}S}{\mathrm{d}t} = \lambda \nabla^2 T', \tag{3}$$

where $\mathrm{d}/\mathrm{d}t = \partial/\partial t + (v\nabla)$, and all the parameters have their obvious meaning. We restrict our discussion to the linear analysis which justifies the neglect of $(v\nabla)v$ term in Eq. (2) as well as the viscous dissipation and the viscous stress tensor in hydrodynamic equations.

The temperature T', velocity v', pressure p', and density ρ' are perturbed around their equilibrium values

$$T' = \overline{T} + T_0(z) + T, \quad p' = \overline{p} + p_0(z) + p, \quad \rho' = \overline{\rho} + \rho_0(z) + \rho, \quad v' = 0 + v, \tag{4}$$

where in the presence of gravity and the imposed temperature gradient,

$$\nabla T_0 = -A, \quad \nabla p_0 = -\overline{\rho} g, \quad \nabla \rho_0 = -\beta \overline{\rho} \nabla T_0 + \alpha_T \nabla p_0 = \overline{\rho} \beta A - \alpha_T \overline{\rho} g \tag{5}$$