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Vehicular traffic through a self-similar sequence of traffic lights

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Abstract

We study the dynamical behavior of vehicular traffic through a sequence of traffic lights positioned self-similarly on a highway, where all traffic lights turn on and off simultaneously with cycle time T_s. The signals are positioned self-similarly by Cantor set. The nonlinear-map model of vehicular traffic controlled by self-similar signals is presented. The vehicle exhibits the complex behavior with varying cycle time. The tour time is much lower such that signals are positioned periodically with the same interval. The arrival time $T(x)$ at position x scales as $(T(x)-x) \propto x^{d_f}$, where d_f is the fractal dimension of Cantor set. The landscape in the plot of $T(x)-x$ against cycle time T_s shows a self-affine fractal with roughness exponent $\alpha = 1-d_f$.

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1. Introduction

Mobility, nowadays, is one of the most significant ingredients of a modern society. Recently, traffic flow, pedestrian flow, and bus-route problems have been studied from the point of view of statistical mechanics and nonlinear dynamics [1–23]. Transportation problems have attracted much attention in the fields of physics [1–5]. Interesting dynamical behaviors have been found in the transportation system. The jams and chaos are typical signatures of the complex behavior of transportation [24,25].

In urban traffic, vehicles are controlled by traffic lights to give priority for a road because the city traffic networks often exceed the capacity. Brockfeld et al. [26] have studied optimizing traffic lights for city traffic by using a CA traffic model . They have clarified the effect of signal control strategy on vehicular traffic. They have also shown that the city traffic controlled by traffic lights can be reduced to a simpler problem of a single-lane highway. Sasaki and Nagatani [27] have investigated the traffic flow controlled by traffic lights on a single-lane roadway by using the optimal velocity model . They have derived the relationship between the road capacity and jamming transition.

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Vehicular traffic depends highly on both signal configuration and cycle time, where the cycle time is the period of a traffic light and the split of signal is the fraction of green time to the signal period. Until now, one has studied the periodic traffic controlled by a few traffic lights. It has been concluded that the periodic traffic does not depend on the number of traffic lights [26,27]. Very recently, a few works have been done for the traffic of vehicles moving through an infinite series of traffic lights with the same interval. The effect of cycle time on vehicular traffic has been classified [28–33]. In real traffic, the vehicular traffic depends highly on the configuration of traffic lights. However, the effect of signal configuration on traffic has not been investigated for the vehicular traffic through a series of signals. The signal configuration is very complex. There exist many signals within a city, while the number of signals is very sparse on a highway. The interval between signals within a city is much shorter than that on a highway. In short-length scale within the city, the configuration of signals is roughly periodic, while it is non-periodic in the long-length scale far from the city. The distribution of signal's intervals may be scale-free for large space.

In this paper, we study the traffic of vehicles moving through a series of traffic lights positioned self-similarly (Cantor set). We present a nonlinear-map model for traffic through the self-similar series of signals. We investigate the dynamical behavior of a single vehicle by iterating the nonlinear map. We clarify the dynamical behavior of a single vehicle through a self-similar sequence of signals by varying cycle time. We compare the vehicular traffic with that through a periodic sequence of signals. We also study the scaling behavior of vehicular traffic.

2. Nonlinear-map model

We consider the flow of vehicles going through the self-similar series of traffic lights. Each vehicle passes freely over other vehicles. Thus, each vehicle does not depend on the other and is uncorrelated with the other vehicles. Therefore, we consider the dynamical behavior of a single vehicle. The traffic lights are self-similarly positioned according to Cantor set [34,35]. The minimal interval between signals is set as distance l. The vehicle moves with the mean speed v between a traffic light and its next light. The traffic lights are numbered, from upstream to downstream, by 1,2,3, …, n, $n+1$, …. The position x_n of signal $n = 1, 2, 3, \ldots$ is given by $x_1 = 0$, $x_2 = l$, $x_3 = 2l$, …. The position x_m of signal $m = 2^k$ is given by $x_m = 3^{k-1}l$. Generally, the position x_{m+i} of signal $m+i$ ($i = 1,2,3\ldots,m$) is given by

$$x_{m+i} = 2 \times 3^{k-1}l + x_i \quad \text{for } m = 2^k \text{ and } 1 \leqslant i \leqslant m. \tag{1}$$

In the synchronized strategy, all the traffic lights change simultaneously from red (green) to green (red) with a fixed time period $(1-s_p)t_s$ (s_pt_s). The period of green is s_pt_s and the period of red is $(1-s_p)t_s$. Time t_s is called the cycle time and fraction s_p represents the split which indicates the ratio of green time to cycle time. We set $s_p = 0.5$. When a vehicle arrives at a traffic light and if the traffic light is red, the vehicle stops at the position of the traffic light. Then, when the traffic light changes from red to green, the vehicle goes ahead. On the other hand, when a vehicle arrives at a traffic light and if the traffic light is green, the vehicle does not stop and goes ahead without changing speed.

We define the arrival time of the vehicle at traffic light n as $t(n)$. The arrival time at traffic light $n+1$ is given by

$$t(n+1) = t(n) + (x_{n+1} - x_n)/v + (r(n) - t(n))H(t(n) - (\mathrm{int}(t(n)/t_s)t_s) - t_s/2)$$
$$\text{with } r(n) = (\mathrm{int}(t(n)/t_s) + 1)t_s, \tag{2}$$

where $H(t)$ is the Heaviside function: $H(t) = 1$ for $t \geqslant 0$ and $H(t) = 0$ for $t < 0$. $H(t) = 1$ if the traffic light is red, while $H(t) = 0$ if the traffic light is green. $(x_{n+1}-x_n)/v$ is the time it takes for the vehicle to move between lights n and $n+1$. $r(n)$ is such time that the traffic light just changed from red to green. The third term on the right-hand side of Eq. (2) represents such time that the vehicle stops if traffic light n is red. The number n of iteration increases one by one when the vehicle moves through the traffic light. The iteration corresponds to the going ahead on the highway.