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Yet on statistical properties of traded volume: Correlation and mutual information at different value magnitudes

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Abstract

In this article we analyse linear correlation and non-linear dependence of traded volume, v, of the 30 constituents of Dow Jones Industrial Average at different value scales. Specifically, we have raised v to some real value α or β, which introduces a bias for small ($\alpha, \beta < 0$) or large ($\alpha, \beta > 1$) values. Our results show that small values of v are regularly *anti-correlated* with values at other scales of traded volume. This is consistent with the high liquidity of the 30 equities analysed and the asymmetric form of the multi-fractal spectrum for traded volume which has supported the dynamical scenario presented by us.

Keywords: Financial market; Traded volume; Correlation; Mutual information; Non-extensive statistical mechanics

1. Introduction

Financial market analysis has become one of the most significative examples about application of concepts associated with physics to systems that are usually studied by other sciences [1]. In this sense, ideas like *scale invariance* and *cooperative phenomena* have also found significance in systems that are not described neither by some Hamiltonian nor some other kind of equation usually associated with Physics (e.g., a master equation). Although plenty of work has been made on the analysis and mimicry of price fluctuations, less attention has been paid to an important observable intimately related to changes in price, the *traded volume*, v [2]. In fact, traded volume has been coupled to price fluctuations whether on an empirical or analytical way for some time [3]. Nonetheless, a consistent analysis of intrinsic statistical properties of traded volume appears to be first presented in Ref. [4]. Thereafter, it has been enlarged or revisited by different authors [5–8]. In this article, we apply a generalisation of the traditional linear self-correlation function in order to study how small, large, and about average (henceforth referred to as frequent) values of v relate between them in time. Furthermore, we analyse non-linear dependence using a generalised measure based on Kullback–Leibler (KL) mutual information. Our data set is made up of 1 min traded volume time series, running from the 1st July 2004 to the 31st December 2004, of the 30 equities that make the Dow Jones Industrial Average index. Aiming to avoid the well-known intraday profile, traded volume time series were previously treated according to a standard procedure (see e.g., Ref. [8]).

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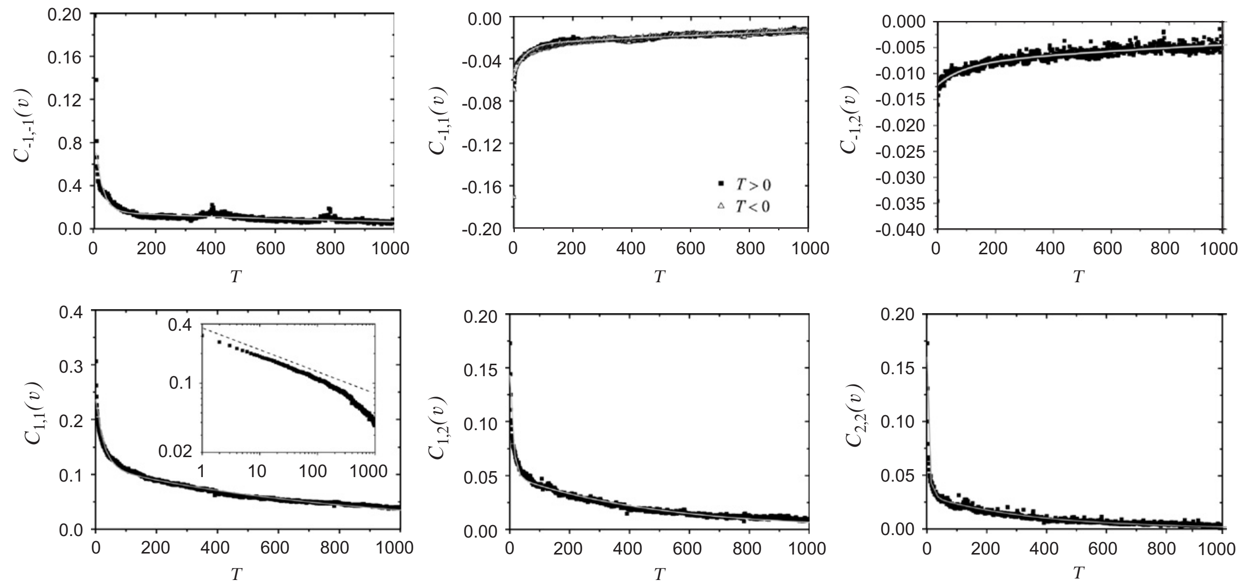

Fig. 1. $C_{\alpha,\beta}(v)$ vs. T for the values of α and β presented in Table 1 (clockwise). On each panel black symbols are the values obtained from time series and the grey line represents the numerical fit of $C_{\alpha,\beta}(v)$ for a double exponential. In panel for $C_{-1,1}(v)$ the curves for $T>0$ and $T<0$ concur, which goes along the lines of time symmetry. The inset on $C_{1,1}(v)$ panel is a log–log representation of the main panel. As it is visible the correlation function does not present power-law behaviour. The same happens for all the other values of (α, β) studied.

2. Generalised linear self-correlation function

The (normalised) correlation function, generally,

$$C(A(\mathbf{r},t), B(\mathbf{r}',t')) = \frac{\langle A(\mathbf{r},t)B(\mathbf{r}',t')\rangle - \langle A(\mathbf{r},t)\rangle\langle B(\mathbf{r}',t')\rangle}{\sqrt{\langle A(\mathbf{r},t)^2\rangle - \langle A(\mathbf{r},t)\rangle^2}\sqrt{\langle B(\mathbf{r}',t')^2\rangle - \langle B(\mathbf{r}',t')\rangle^2}}, \tag{1}$$

represents a useful analytical form to evaluate how much two random variables depend, linearly, on each other. Leaving out spatial dependence, when A and B are the same observable, Eq. (1) represents the straightforwardest way to appraise memory in the evolution of A. In any case, it does not give us any information about the role of magnitudes. Inspired by multi-fractal analysis [9], a simple way to quantify this type of correlation can be defined by introducing a *generalised self-correlation function*,

$$C(\hat{A}(t), \tilde{A}(t')) \equiv C_\alpha, \beta(A),$$

where $\hat{A}(t) = |A(t)|^\alpha$, $\tilde{A}(t) = |A(t)|^\beta$ (with $\alpha, \beta \neq 0 \in \Re$), and $t' = t + T$.[1] As an example let us assume $\beta = 1$. For values of α greater than 1, small values of A become even smaller and their weight in the value of $C_{\alpha,\beta}(A)$, due to $\hat{A}(t), \tilde{A}(t')$, approaches negligibility (e.g., when $\alpha = 2$, $v = 10^{-1} > v^\alpha = 10^{-2}$ and $v = 10 < v^\alpha = 10^2$). Otherwise, when α is negative, we highlight values around zero (e.g., when $\alpha = -1$, $v = 10^{-1} < v^\alpha = 10^1$ and $v = 10 > v^\alpha = 10^{-1}$). In the end, after summing over all pairs $(\hat{A}(t), \tilde{A}(t'))$, we verify that the main contribution for $C_{\alpha,1}(A)$ comes from large values of $|A(t)|$ when $\alpha > 1$ and from small values of $|A(t)|$ when $\alpha < 0$. Accordingly, for $\alpha = \beta$, we estimate how values of the same order of magnitude are related in time, when $\alpha \neq \beta$ we analyse the relation between values with different magnitudes.

In Fig. 1 we depict the results that we have obtained by applying Eq. (1), with different pairs of (α, β) in traded volume time series. In Table 1 we present the values of the numerical adjustment of $C_{\alpha,\beta}(A)$ for a

[1] Hereinafter $A(t)$ is assumed to be a stationary time series. The dependence on the *waiting time*, t represents an indication of non-stationarity in the signal.