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True and apparent scaling: The proximity of the Markov-switching multifractal model to long-range dependence

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Abstract

In this paper, we consider daily financial data of a collection of different stock market indices, exchange rates, and interest rates, and we analyze their multi-scaling properties by estimating a simple specification of the Markov-switching multifractal (MSM) model. In order to see how well the estimated model captures the temporal dependence of the data, we estimate and compare the scaling exponents $H(q)$ (for $q = 1, 2$) for both empirical data and simulated data of the MSM model. In most cases the multifractal model appears to generate 'apparent' long memory in agreement with the empirical scaling laws.

Keywords: Scaling; Generalized Hurst exponent; Multifractal model; GMM estimation

1. Introduction

The scaling concept has its origin in physics but it is increasingly applied outside its traditional domain. In the literature [1–3] different methods have been proposed and developed in order to study the multi-scaling properties of financial time series. For more details on scaling analysis see Ref. [4].

Going beyond the phenomenological scaling analysis, the multifractal model of asset returns (MMAR) introduced by Mandelbrot et al. [5] provides a theoretical framework that allows to replicate many of the scaling properties of financial data. While the practical applicability of MMAR suffered from its combinatorial nature and its non-stationarity, these drawbacks have been overcome by the introduction of iterative multifractal models (Poisson MF or Markov-switching multifractal (MSM) model [6–8]) which preserve the hierarchical, multiplicative structure of the earlier MMAR, but are of much more 'well-behaved' nature concerning their asymptotic statistical properties. The attractiveness of MF models lies in their ability to mimic the stylized facts of financial markets such as outliers, volatility clustering, and asymptotic power-law behavior of autocovariance functions (long-term dependence). In contrast to other volatility models with long-term dependence [9], MSM models allow for multi-scaling rather than uni-scaling with varying decay

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exponents for all powers of absolute values of returns. One may note, however, that due to the Markovian nature, the scaling of the Markov-switching MF model only holds over a limited range of time increments depending on the number of hierarchical components and this 'apparent' power-law ends with a cross-over to an exponential cut-off.

With this proximity to true multi-scaling, it seems worthwhile to explore how well the MSM model could reproduce the empirical scaling behavior of financial data. To this end, we estimate the parameters of a simple specification of the MSM model for various financial data, and we assess its ability to replicate the empirical scaling behavior by also computing $H(q)$ by means of the generalized Hurst exponent (GHE) approach [4,10,11] and H by means of the modified R/S method [12] for the same data sets. We then proceed by comparing the scaling exponents for empirical data and simulated time series based on our estimated MSM models. As it turns out, the MSM model with a sufficient number of volatility components generates pseudo-empirical scaling laws in good overall agreement with the empirical results.

The structure of the paper is as follows: In Section 2 we introduce the multifractal model, the GHE, and the modified R/S approach. Section 3 reports the empirical and simulation-based results. Concluding remarks and perspectives are given in Section 4.

2. Methodology

2.1. Markov-switching multifractal model

In this section, we shortly review the building blocks of the MSM process. Returns are modeled as [7,8]

$$r_t = \sigma_t \cdot u_t, \tag{1}$$

with innovations u_t drawn from a standard normal distribution $N(0,1)$ and instantaneous volatility being determined by the product of k volatility components or multipliers $M_t^{(1)}, M_t^{(2)}, \ldots, M_t^{(k)}$ and a constant scale factor σ:

$$\sigma_t^2 = \sigma^2 \prod_{i=1}^{k} M_t^{(i)}. \tag{2}$$

In this paper, we choose, for the distribution of volatility components, the binomial distribution: $M_t^{(i)} \sim \{m_0, 2-m_0\}$ with $1 \leqslant m_0 < 2$. Each volatility component is renewed at time t with probability γ_i depending on its rank within the hierarchy of multipliers and it remains unchanged with probability $1-\gamma_i$. The transition probabilities are specified by Calvet and Fisher [7] as

$$\gamma_i = 1-(1-\gamma_k)^{(b^{i-k})}, \quad i = 1, \ldots, k, \tag{3}$$

with parameters $\gamma_k \in [0,1]$ and $b \in (1,\infty)$. Different specifications of Eq. (3) can be arbitrarily imposed (cf. [8] and its earlier versions). By fixing $b=2$ and $\gamma_k = 0.5$, we arrive a relatively parsimonious specification:

$$\gamma_i = 1-(1-\gamma_k)^{(2^{i-k})}, \quad i = 1, \ldots, k. \tag{4}$$

This specification implies that replacement happens with probability of one half at the highest cascade level. Various approaches have been employed to estimate multifractal models. The parameters of the combinatorial MMAR have been estimated via an adaptation of the scaling estimator and Legendre transformation approach from statistical physics [13]. However, this approach has been shown to yield very unreliable results [14]. A broad range of more rigorous estimation methods have been developed for the MSM model. Calvet and Fisher [6] propose maximum likelihood estimation, while Lux [8] proposes a generalized method of moments (GMM) approach, which can be applied not only to discrete but also to continuous distributions of the volatility components. In this paper, GMM is used to estimate the two MSM model parameters in Eq. (2), namely: $\hat{\sigma}$ and $\hat{m}_0$.