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Frequency analysis of tick quotes on the foreign exchange market and agent-based modeling: A spectral distance approach

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Abstract

High-frequency financial data of the foreign exchange market (EUR/CHF, EUR/GBP, EUR/JPY, EUR/NOK, EUR/SEK, EUR/USD, NZD/USD, USD/CAD, USD/CHF, USD/JPY, USD/NOK, and USD/SEK) are analyzed by utilizing the Kullback–Leibler divergence between two normalized spectrograms of the tick frequency and the generalized Jensen–Shannon divergence among them. The temporal structure variations of the similarity between currency pairs is detected and characterized. A simple agent-based model in which N market participants exchange M currency pairs is proposed. The equation for the tick frequency is approximately derived theoretically. Based on the analysis of this model, the spectral distance of the tick frequency is associated with the similarity of the behavior (perception and decision) of the market participants in exchanging these currency pairs.
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1. Introduction

The recent accumulation of high-frequency financial data due to the development and spread of information and communications technology has sparked interest in financial markets [1–10]. Many researchers expect new findings and insights into the worlds of both finance and physics. Since the financial markets are complex systems that consist of several agents that interact with one another, an enormous amount of data must be treated in order to describe and understand them at the microscopic level. Therefore, it is important to find adequate variables or relevant quantities to describe their properties [11]. Since a macroscopic description allows information with global properties to be compressed, if the adequate macroscopic quantities can be determined, then relationships can be established among various macroscopic quantities and a deeper understanding of the system can be obtained.

On the other hand, agent-based models as complex systems are attracting significant interest across a broad range of disciplines. Several agent-based models have been proposed to explain the behavior of financial markets during the last decade [12–18]. Agent-based models are expected to provide an alternative to

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phenomenological models that mimic market price fluctuations. Specifically, it seems to be worth considering the explanation capability of the agent-based models for causality from a microscopic point of view.

In a previous study, the tick frequency, which is defined as the number of tick quotations per unit time, was reported to exhibit periodic motions due to the collective behavior of the market participants [19,20]. As a result, the tick frequency appears to be an important representative quantity in the financial market. Moreover, it has been reported that it is possible to detect the dynamic structure of the foreign exchange market by using the spectral distance defined by the Kullback–Leibler divergence [21]. The spectral distance of the tick frequency is one possibility for macroscopically describing the relationship among market participants in the financial market.

In the present study, the meaning of the spectral distance of the tick frequency is discussed, starting from the microscopic description with the agent-based model of a financial market. First, definitions and the results of the spectral distance of the tick frequency are presented. Next, a model that consists of N market participants who choose their action among three kinds of investment attitudes in order to exchange M currency pairs is considered. In this model, the heterogeneous agents perceive information in the environment, which is separated into exogenous factors (news about events) and endogenous factors (news about market fluctuations), and decide their actions based on these factors. There are two thresholds by which to select their actions among three kinds of investment attitudes (buying, selling, and waiting). Analysis of this model indicates that the spectral distance of the tick frequency is equivalent to the difference among behavioral parameters of market participants who exchange these currency pairs.

This remainder of this paper is organized as follows. In Section 2, the tick frequencies of 12 currency pairs in the foreign exchange market are analyzed with the spectral distance measured by the Kullback–Leibler divergence and the Jensen–Shannon divergence of the normalized power spectra. In Section 3, an agent-based model in which N market participants deal with M currency pairs is proposed. In Section 4, based on the agent-based model, the equation for the tick frequency is approximately derived, and the relationship between the spectral distance of the tick frequency and the behavioral parameters of market participants is discussed. Section 5 is devoted to concluding remarks.

2. Frequency analysis

2.1. Data

The foreign currency market data of United States Dollar (USD), Euro (EUR), Switzerland Francs (CHF), Great Britain Pounds (GBP), Norwegian Krone (NOK), Swedish Krona (SEK), Canadian Dollars (CAD), New Zealand Dollars (NZD), and Japanese Yen (JPY), as provided by CQG Inc., were investigated [22]. The data include two quote rates, namely, the bid rate and the ask rate, with a resolution of 1 min. The bid and ask rates are the prices at which bank traders are willing to buy and sell a unit of currency. All traders in the foreign exchange market have a rule to quotes both rates at the same time (two-way quotation). Generally, the ask rate is higher than the bid rate, and the difference between the bid rate and the ask rate is called the bid–ask spread.

The data investigated in this article are from two databases. The first includes 12 currency pairs, EUR/CHF, EUR/GBP, EUR/JPY, EUR/NOK, EUR/SEK, EUR/USD, NZD/USD, USD/CAD, USD/CHF, USD/JPY, USD/NOK, and USD/SEK, during the period from the 1st to the 29th of September 2000. The other includes three currency pairs, EUR/USD, EUR/JPY, and USD/JPY, during the period from the 4th of January 1999 to the 31st of December 2004. The data start at 17:00 (CST) on Sunday, and finish at 16:59 (CST) on Friday. The foreign exchange market is open for 24 h on weekdays. On Saturdays and Sundays, there are no ticks on the data set because most banks are closed.

2.2. Methods

The tick frequency is defined by counting the number of times that bank traders quote the bid and ask rates per unit time. According to this definition a currency pair having a high (low) quote frequency indicates activity (inactivity). Since bank traders usually quote both bid and ask rates at the same time, it is sufficient to