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Anderson localization problem: An exact solution for 2-D anisotropic systems

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Abstract

Our previous results [V.N. Kuzovkov, W. von Niessen, V. Kashcheyevs, O. Hein, J. Phys. Condens. Matter 14 (2002) 13777] dealing with the analytical solution of the two-dimensional (2-D) Anderson localization problem due to disorder is generalized for anisotropic systems (two different hopping matrix elements in transverse directions). We discuss the mathematical nature of the metal–insulator phase transition which occurs in the 2-D case, in contrast to the 1-D case, where such a phase transition does not occur. In anisotropic systems two localization lengths arise instead of only one length.

Keywords: Random systems; Anderson localization; Phase diagram

1. Introduction

Anderson localization [1] remains one of the main problems in the physics of disordered systems (see e.g., the review articles [2–4]). In the series of our previous papers [5–7] we presented an exact analytic solution to this problem. By the exact solution we mean the calculation of the phase diagram for the metal–insulator system. We have been able to solve the two dimensional (2-D) problem [5]. We have shown then that the phase of delocalized states exists for a non-interacting electron system. The main aim of the paper [6] was the generalization of the results to the case of higher dimensional spaces (N-D). In Ref. [7] we discussed the mathematical details of the new analytical approach for calculating the phase diagram. An exact solution is only possible for the conventional Anderson model: the tight-binding approximation with diagonal disorder, where on-site potentials are independently and identically distributed.

It is well known that the exact results in the field of phase transitions (the metal–insulator transition is a particular case of a phase transition) are exceedingly rare [8,9]. This is why any extension of the applicability

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range of analytical methods in this field is of great interest. In this paper we extend our approach [5–7,10] to *anisotropic* media [11–14], but else remaining in the framework of a conventional Anderson model.

Incorporation of an anisotropy into the tight-binding approximation with diagonal disorder is also methodologically valuable. Before the exact solution was obtained for $D>1$ [5–7], analytical methods concentrated on the $D=1$ case [15,16]. However, the specific topology of 1-D systems does not permit the extension of the results to higher dimensions. This is true in particular for the Ising model, where an exact analytical solution for the 1-D case has nothing to do with the 2-D solution obtained by Onsager [8,9]. As it is well known for the Anderson localization problem, all states in the 1-D system are localized (i.e., there is no metal–insulator transition). This is a particular result of a general theory [9] that no phase transitions are possible in 1-D systems with short-range interactions. In fact, phase transitions (e.g., in the Ising model [8,9] or the Anderson localization problem [5]) are observed only starting with $D=2$.

It should be realized that the approximate methods are also of a limited use here. In particular, traditional perturbation theory works perfectly in the 1-D case [15], which is well demonstrated by the analysis of the exact solution in Refs. [5,17]. Random potentials can be treated in the 1-D Anderson localization problem as a small parameter, which is used in a series expansions of physical quantities. However, this approach fails [9] for systems with phase transitions in $D>1$, since physical quantities here are no longer described by analytical functions and series expansions. The same is valid for the Lyapunov exponent γ (which is the inverse of the localization length ξ in the Anderson problem [5]).

Of particular interest is the understanding of the mathematical nature of the phase transition: how does the analytical character of the exact solution for the 1-D problem change to a non-analytical character of the exact solution for the 2-D case? For the *discrete* spatial dimensions (1-D or 2-D) a simple comparison of the two relevant solutions does not help us. However, a "*continuous*" treatment of the spatial dimension for anisotropic systems, performed in this paper, provides much more insight. We have put the word continuous within quotation marks because we have in the present case of anisotropic systems an interesting possibility of a transition from a 2-D system to a 1-D system, by considering the limit $\kappa \to 0$ for parameter of anisotropy κ.

The paper is organized as follows. In Section 2 we discuss the basic equations for the isotropic problem [5], which are generalized there for the anisotropic case. Section 3 presents the main results for the anisotropic problem. We demonstrate how the study of the limiting case of a strong anisotropy permits us to establish a relation between 1-D and 2-D cases.

2. Recursion relation and the filter function

2.1. Isotropic system

Let us start with the Schrödinger equation for the isotropic system (the lattice constant and the hopping matrix element are set equal to unity)

$$\psi_{n+1,m}+\psi_{n-1,m}+\psi_{n,m+1}+\psi_{n,m-1}=(E-\varepsilon_{n,m})\psi_{n,m}. \tag{1}$$

The on-site potentials $\varepsilon_{n,m}$ are independently and identically distributed with existing first two moments, $\langle\varepsilon_{n,m}\rangle=0$ and $\langle\varepsilon_{n,m}^2\rangle=\sigma^2$.

Eq. (1) can be written as the recursion

$$\psi_{n+1,m}=-\varepsilon_{n,m}\psi_{n,m}-\psi_{n-1,m}+\mathscr{L}\psi_{n,m}, \tag{2}$$

where the operator $\mathscr{L}$ acts on the index m and is defined by the relation

$$\mathscr{L}\psi_{n,m}\equiv E\psi_{n,m}-\sum_{m'=\pm1}\psi_{n,m+m'}. \tag{3}$$

The standard initial (boundary) conditions are $\psi_{0,m}=0$ and $\psi_{1,m}=\alpha_m$, respectively.

The existence of some fundamental 1-D numerical series h_n ($n=0,1,\ldots,\infty$), the so-called system function or *filter*, was proved rigorously in Refs. [5–7]. A study of the asymptotic behaviour of this series allows to define uniquely the *phase diagram* of the system. Namely, the series h_n is bounded

$$|h_n|<\infty, \tag{4}$$