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On feedback and stable price adjustment mechanisms

Guillermo Fernandez-Anaya*, Jose Alvarez-Ramirez[1], Carlos Ibarra-Valdez[1]

Departamento de Fisica y Matematicas, Universidad Iberoamericana, Av. Prolongacion Paseo de la Reforma 880, Mexico D.F. 01210, Mexico

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Abstract

Given an excess demand function of an economy, say $Z(p)$, a stable price adjustment mechanism (SPAM) guarantees convergence of solution path $p(t, p_0)$ to an equilibrium p_{eq} solution of $Z(p) = 0$. Besides, all equilibrium points of $Z(p)$ are asymptotically stable. Some SPAMs have been proposed, including Newton and transpose Jacobian methods. Despite this powerful stability property of SPAMs, their acceptation in the economics community has been limited by a lack of interpretation. This paper focuses on this issue. Specifically, feedback control theory is used to link SPAMs and price dynamics models with control inputs, which match the economically intuitive Walrasian Hypothesis (i.e., prices change with excess demand sign). Under mild conditions, it is shown the existence of a feedback function that transforms the price dynamics into a desired SPAM. Hence, a SPAM is interpreted as a fundamental (e.g., Walrasian) price dynamics under the action of a feedback function aimed to stabilize the equilibrium set of the excess demand function.
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1. Introduction

Consider a continuous-time economy with a continuously differentiable excess demand function $Z(p) \in \mathbb{R}^n$, where $p \in \mathbb{R}^n$ is used to denote the price vector. The equilibrium set of the excess demand function $Z(p)$ is given by $E = \{p_{eq} \in \mathbb{R}^n : Z(p_{eq}) = 0\}$. A basic problem in General Equilibrium Theory has been to construct a vector field $p \mapsto \Omega(p) \in \mathbb{R}^n$ such that, for almost all initial condition $p_0 \in \mathbb{R}^n$, the solution path $p(t, p_0)$ of the differential system $\dot{p} = \Omega(p)$ converges asymptotically to an equilibrium $p_{eq} \in E$. Indeed, all $p_{eq} \in E$ become a locally asymptotically stable equilibrium point of the differential system $\dot{p} = \Omega(p)$, which is called a *stable price adjustment mechanism* (SPAM). The economic importance of this problem relies on the fact that, at an equilibrium $p_{eq} \in E$, demand $D(p)$ matches supply $S(p)$, i.e., $D(p_{eq}) = S(p_{eq})$, and so all market agents are able to fulfill in a stable way their economic expectations formed within a competitive environment [1,2]. In a seminal paper, Smale [3] provided a SPAM by means of a continuous-time version of the well-known Newton

*Corresponding author.
E-mail address: guillermo.fernandez@uia.mx (G. Fernandez-Anaya).
[1]J. Alvarez-Ramirez and C. Ibarra-Valdez are at Division de Ciencias Basicas e Ingenieria Universidad Autonoma Metropolitana-Iztapalapa.

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method for finding all the roots of a nonlinear equation ($Z(p)$ in our case). Specifically, the Smale's Newton method (SNM in the sequel) is given by $\Omega(p) = -\sigma[\partial_p Z(p)]^{-1} Z(p)$, where $\partial_p Z(p)$ is the Jacobian matrix of the excess demand function $Z(p)$ and $\sigma > 0$ is a tuning parameter. In this way, the corresponding price dynamics is given by

$$\dot{p} = -\sigma[\partial_p Z(p)]^{-1} Z(p). \quad (1)$$

It has been proven that each $p_{eq} \in E$ is locally asymptotically stable, so that the path solution $p(t, p_0)$ converges to an equilibrium $p_{eq} \in E$ for all initial condition p_0 in a neighborhood of $N_{eq} \subset \mathbb{R}^n$ [3]. Subsequently, Saari and Simon [4] found that the dependence of the price adjustment mechanism only on excess demand in the appropriate market is not enough, and that the entire Jacobian $\partial_p Z(p)$ needs to be used in order to achieve convergence to an equilibrium price for almost all initial conditions. In fact, the Jacobian matrix contains essential information on the directionality of price changes under excess demand disturbances [5]. This information is incorporated as in Smalés Newton method Eq. (1), the vector field is oriented in a way that all the equilibrium prices are locally stabilized and their region of attraction are separated by singularity manifolds. To the best of our knowledge, at least two modifications to the SNM have been reported in the literature. Kayima [6] proposed a modification of the SNM to account for the effectiveness of an initial guess. A "drawback" of SNM and the Kayima scheme is that their implementation requires Jacobian matrix inversion, which can lead to numerical overflow because of Jacobian matrix singularities. To avoid this situation, recently Mukherji [7] proposed the usage of the transpose Jacobian matrix, so that Jacobian matrix singularities are no longer an implementation issue.

Intuitively, the full Jacobian matrix $\partial_p Z(p)$ seems to be necessary to converge to an equilibrium because it contains the information on the direction of changes of the excess demand with respect to changes in the price vector [9]. In this way, under Jacobian matrix inversion, the individual price dynamics are decoupled modulo some zero measure singularities, and convergence to equilibrium prices can be achieved. Within this reasoning, Saari and Simon [4] showed that the information on the Jacobian matrix $\partial_p Z(p)$ is actually a necessary condition in order to achieve stability of an economic system (represented by the excess demand function $Z(q)$) about its equilibrium set E. On the other hand, there is a series of structural theorems that put limits on the search of reasonable conditions about excess demand functions and, hence, on conditions guaranteeing global stability of tatonnement price adjustments. They are, in increasing complexity order, the Seiber Theorem, the Sonnenschein–Debreu–Mantel Theorem and the Kirman–Koch Theorem, all treating on aggregation over agents [8,9].

There is a plethora of economical models displaying instabilities, from multiplicity of equilibrium prices to complex, presumably chaotic, erratic behavior from rational choices [10,11] and irregular growth cycles from macroeconomic models [12,13] have been reported. Chaotic behavior for continuous-time models has been studied [14]. In particular, the relationship between discrete- and continuous-time representation of economic phenomena, and the combined role of lags and non-linearity in generating chaotic output have been emphasized. It has also been shown that adaptive expectation for simple cobweb economical models can display routes to cyclic and even chaotic behavior [15]. Complicated dynamics for a simple labor market dynamics were also found recently [16]. This sample of economical model results shows that when the market dynamics evolve according to the interaction between demand and supply, instabilities can arise naturally. In particular, it seems that economic operation about equilibrium prices occur seldom, leading to market imbalances. In addition, this also demonstrates a significant gap between unstable economical dynamics and SPAMs. One immediately can highlight the importance of disposing of a SPAM because, in such situation, a stable economy operation about equilibrium conditions can be guaranteed. Indeed, stable operation is a must to ensure the reliability of a socioeconomic system. Despite such an important property of SPAMs, like the SNM, they have received several criticisms in the recent two decades. A major basic criticism is its lack of economic interpretation. In fact, the economic intuition behind the convergence of solutions to an equilibrium is not always clear since (i) price dynamics in SPAMs follows the Walrasian Hypothesis, and (ii) economic models whose price dynamics follows the Walrasian Hypothesis can be unstable. This lack of economic foundations is maybe due to the fact that, for instance, the SNM was derived on the basis of pure mathematical arguments (e.g., to find all the roots of the nonlinear function $Z(p)$) without considerations of economic appealing [8]. This paper focuses in this issue; namely, to gain some insights on the link between

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