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Is non-Gaussianity sufficient to produce long-range volatile correlations?

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Abstract

Scaling analysis of the magnitude series (volatile series) has been proposed recently to identify possible non-linear/multifractal signatures in the given data [Y. Ashkenazy, et al. Phys. Rev. Lett. 86 (2001) 1900; Y. Ashkenazy, et al. Physica A 323 (2003) 19; T. Kalisky, Y. Ashkenazy, S. Havlin. Phys. Rev. E 72 (2005) 011913]. In this article, correlations of volatile series generated from stationary first-order linear feedback process with Gaussian and non-Gaussian innovations are investigated. While volatile correlations corresponding to Gaussian innovations exhibited uncorrelated behavior across all time scales, those of non-Gaussian innovations showed significant deviation from uncorrelated behavior even at large time scales. The results presented raise the intriguing question whether non-Gaussian innovations can be sufficient to realize long-range volatile correlations.

Keywords: Detrended fluctuation analysis; Volatility analysis

1. Introduction

Detrended fluctuation analysis (DFA) and its extensions [1–6] have been used widely to determine the nature of correlations in synthetic and experimental data obtained from a wide range of complex systems. Recently [1–3], analysis of the magnitude series of the given empirical sample has been used to gain further insight into the underlying dynamics [5–10]. More importantly, long-range correlation in the magnitude series was found to be indicative of non-linear and possibly multifractal signatures in the given data [1–3, 5–10]. Several models have also been proposed recently to generate volatile correlations under certain constraints [11]. In the present study, we investigate the impact of Gaussian and non-Gaussian innovations on the scaling of magnitude series generated from stationary first-order linear feedback processes.

2. Methods

A first-order linear feedback process represents the most elementary of the stochastic processes, and is given by the expression

$$x_n = \theta x_{n-1} + \epsilon_n, \tag{1}$$

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where $\in_n$ represents an identical and independently distributed (i.i.d.) process (white noise) sampled from a given distribution, also known as *innovations*. The x_n^{th} sample is related to the x_{n-1}^{th} sample through the process parameter θ. For the same reason, processes such as Eq. (1) are termed finite memory or Markov processes. Each x_n is a weighted sum or a linear combination of innovations $\in_n$. Therefore, Eq. (1) is a *linearly correlated noise* whose distribution is governed by $\in_n$. It can be shown analytically that the above process (1) is stationary for $|\theta|<1$ with associated auto-correlation function $\rho(k) = \theta^k$ (see Appendix A). In the present study, we considered process parameters $\theta = 0.95$ and $\theta = 0.65$. Unlike the latter, $\theta = 0.95$ is close to the non-stationary regime $\theta = 1$, resulting in slow decay of the auto-correlation function. We investigate the scaling behavior of Eq. (1) with Gaussian as well as non-Gaussian innovations $\in_n$ sampled from five different distributions, namely,

NORM: Zero-mean unit variance innovations$\in_n$, sampled from a Gaussian-distributed white noise (g) with a probability density function $f(g) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}, \quad x \in (-\infty, \infty)$.
SQNORM: Zero-mean unit variance innovations $\in_n$, sampled from a squared transform of a Gaussian-distributed white noise, i.e., $g_1 = g^2$.
EXPNORM: Zero-mean unit variance innovations $\in_n$, sampled from an exponential transform of a Gaussian-distributed white noise, i.e., $g_2 = e^g$.
UNI: Zero-mean unit variance innovations $\in_n$, sampled from a uniformly distributed white noise (u) with a probability density function $f(u) = 1/(b-a), \quad u \in (a, b)$.
LOGUNI: Zero-mean unit variance innovations $\in_n$, sampled from a negative log transform of a uniformly distributed white noise, i.e., $u_1 = -\log(u)$.

The above abbreviations shall be used in the subsequent sections. It can be shown analytically that second-order moments are sufficient to completely describe first-order linear feedback processes with Gaussian innovations. However, this is not true in the case of non-Gaussian innovations, where higher-order statistics are required to sufficiently describe the process. Two popular statistics used in the literature to reflect the deviation from Gaussianity are skewness (ψ) and kurtosis (κ). Skewness and kurtosis of the innovations (NORM, EXPNORM, SQNORM, UNI and LOGUNI) are shown in Figs. 1a–e. Those of their corresponding linear feedback processes with parameters ($\theta = 0.95$, $N = 2^{16}$) are shown in Figs. 1f–j, respectively. While NORM and UNI are symmetric distributions ($\psi = 0$), SQNORM, EXPNORM and LOGUNI are asymmetric ($\psi \neq 0$). Kurtosis of UNI ($\kappa = 1.8$) and NORM ($\kappa = 3$) are dissimilar (Figs. 1a and d); however, those of their corresponding linear feedback processes are similar ($\kappa = 3$) (Figs. 1f and i). As shall be shown later, volatile correlations of Eq. (1) with UNI showed minimal discrepancy from those with NORM.

3. Results

Classical power spectral analysis is used widely to investigate correlations in stationary linear processes such as Eq. (1). The power spectrum of a stationary process is related to its auto-correlation function by the Wiener–Khinchin theorem. As noted earlier (Appendix A), the expression of the auto-correlation for the first-order linear feedback process x_n (1) is governed solely by the process parameter θ and is immune to the distribution of the innovations $\in_n$. Thus, it might not be surprising to note that first-order linear feedback processes with Gaussian (NORM) and non-Gaussian (SQNORM, EXPNORM, UNI and LOGUNI) innovations revealed similar spectral signatures (Fig. 2a), also reflected in the scaling of their fluctuation function $F(s)$ with time scale s (Fig. 3a), obtained using DFA with fourth-order polynomial detrending. In the literature, volatility series of the given data has been generated using continuous and discontinuous, static, memoryless non-linear transforms [3]. The power spectrum of the magnitude series generated from mean subtracted x (1) with transforms $|x|$, x^2, $|x_n - x_{n-1}|$, and innovations NORM, SQNORM, EXPNORM, UNI and LOGUNI is shown in Fig. 2. As expected, the qualitative behavior of the power spectrum, hence the correlation, showed a marked change across the three different transforms [12]. More importantly, the transforms $|x|$, x^2 do not exhibit considerable variation across the various innovations (Figs. 2b and c), also reflected in the scaling of their fluctuation function $F(s)$ with respect to the time scale s (Figs. 3b and c),