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Are costs of capital necessarily constant over time and across states of nature? Some remarks on the debate on 'WACC is not quite right'

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ABSTRACT

Miller (2009a) opened a debate in this journal on the correct determination of weighted average costs of capital (WACC). So far Bade (2009), Pierru (2009a), Lobe (2009) as well as Keef, Khaled, and Roush (2012) have contributed to this debate. Even though they discuss the same, rather simple valuation problem, the dispute cannot be considered resolved. Whilst they agree that Miller erroneously assumed constant leverage ratios, the center of discussion is now placed on the question whether or not cost of capital is constant over time when leverage changes and interest paid is not tax deductible. In particular, Keef et al. (2012) demand time-invariant WACC and criticize Bade (2009) and Pierru (2009a) for allowing WACC to change over time. The aim of this paper is twofold. Firstly, we show that the arguments of Keef et al. (2012) are flawed and their criticism of Bade (2009) and Pierru (2009a) is thus unfounded. Keef et al. (2012) are wrong to ignore that not only financial risk but also operational risk can change over time. Secondly, we provide evidence that cost of capital can also be dependent on the future state of nature. So far this fact has been neglected by all contributors to this debate and becomes obvious only if state-dependent cash flow realizations, not only their expected values, are considered as well.

1. Introduction

Recently this journal hosted a heated debate on how the weighted average cost of capital is to be interpreted and how it should advantageously be determined. Miller (2009a) argued that a simple linear interpolation of cost of equity and cost of debt leads to wrong valuation results. To prove his claim he presented an extensive example.

Bade (2009) as well as Pierru (2009a) object to Miller vehemently and point out that his valuation approach contains an error. Miller (2009a) wrongly assumed that the leverage ratio is constant over time. Based on the same example used by Miller (2009a) and disregarding taxes, Bade (2009) and Pierru (2009a) show that this leverage ratio does change over time and that hence Miller's result cannot be upheld. In particular, they illustrate that the weighted average cost of capital (WACC) must increase over time when cost of levered equity and of debt remain constant and Miller's repayment schedules apply, i.e. leverage declines. Thus, WACC should be time-dependent. Alternatively, Pierru (2009a) adjusts the repayment schedules assumed in Miller's example in such a manner that the debt ratio as well as WACC remain constant.[2] In his reply Miller (2009b) admits that his original repayment schedule is not compatible with a constant debt ratio and additionally proposes another three (non-self-amortizing) repayment schedules.[3]

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[2] Similarly, Lobe (2009) presents the identical two alternatives for proving the correctness of the traditional WACC. The latter alternative was heavily criticized by Tanha & Foroutan (2013) who base their argumentation on the distinction between the so-called *Total Cash Flow* approach that considers the tax shield in the numerator (see, e.g., (Kruschwitz & Löffler, 2006, Section 2.4.2)) and the *WACC* approach that accounts for the tax shield in the denominator. They state that Pierru (2009a), Bade (2009), and Miller (2009a) "failed to answer Miller's question correctly because they did not consider this fact that they were assuming one description of the cash flow while using another formula for WACC which assumes the other description of the cash flow" (Tanha & Foroutan, 2013, p. 2083). However, the distinction between both approaches (i.e. cash flow descriptions and WACC formulas) is by no means suitable to demonstrate weaknesses in the afore-mentioned papers that all assume a world without taxes where both approaches inevitably coincide.

[3] Also, in his reply Miller claims that the only relevant costs of capital and debt ratios are those that exist at $t=0$, the time the project is accepted and financed. However, the project's present value is generally calculated according to $\frac{E[\widetilde{CF}_1]}{1+WACC_0} + \frac{E[\widetilde{CF}_2]}{(1+WACC_0)(1+WACC_1)} + \ldots + \frac{E[\widetilde{CF}_n]}{(1+WACC_0)\ldots(1+WACC_{n-1})}$. Thus, apart from the special case of

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Recently Keef, Khaled, and Roush (2012) have attempted to summarize the discussion and place it in a wider context. From their point of view, Bade (2009) as well as Pierru (2009a) have made another hitherto hidden mistake and therefore the question raised by Miller remains unsolved. Whilst Keef et al. (2012) agree that leverage indeed decreases in Miller's example, they claim that Bade (2009) and Pierru (2009a) "incorrectly conclude that the annual WACC increases over time".[4] Instead they argue that in a world without taxes WACC is independent of leverage and should thus be constant over time. They believe that Bade (2009) and Pierru (2009a) erroneously "assume, for convenience, that the required rate of return on levered equity is independent of leverage".[5] Analogously, this critique applies to Lobe (2009) whose argumentation is in line with Bade (2009) and Pierru (2009a).

The aim of our study is twofold. Firstly, we are convinced that the arguments of Keef et al. (2012) supporting time-independency of WACC are flawed. Whilst Keef et al. (2012) solely focus on changes in financial risk (resulting from changes in leverage) they neglect the fact that operational risk can change over time, too. In particular, we show that even regardless of taxes WACC can indeed be time-dependent and that the criticism voiced by Keef et al. (2012) is thus ill-founded.

Secondly, we want to draw the reader's attention to another issue that has not been discussed in any of the previously cited papers with sufficient care. Remarkably, all contributors to this debate have ignored the way risk affects cash flows. If, however, one properly accounts for the fact that cash flows are state-contingent, we show that cost of capital does not necessarily have to be independent of future states of nature. This, however, is the implicit assumption in all previously cited papers.

2. Time-dependency of cost of capital

The focus of this section is to analyze the time-dependency of WACC from a theoretical point of view. To illustrate our argumentation and improve its comparability to prior papers in this field we refer to the same example introduced by Miller (2009a) and taken up by Bade (2009), Pierru (2009a), Lobe (2009) as well as Keef et al. (2012). Table 1 in the appendix summarizes the setting. Table 2 reflects the proposed cost of capital that differs according to whether or not WACC has to be constant over time. In line with the proposal of Bade (2009), Pierru (2009a), and Lobe (2009), we show that WACC can indeed change over time and the criticism of Keef et al. (2012) is thus unfounded. This holds true even in absence of taxes – a setting in which WACC equals the cost of unlevered equity r_e^U and is thus independent of leverage. However, it is wrong to assume that r_e^U (a measure for the operational risk to which a firm is exposed) has to be constant over time. Note that in line with all other papers contributing to this discussion we do not explicitly account for interest rate risk. However, the modeling of interest rate changes over time would affect costs of capital which again supports time-varying WACC.

In contrast to Bade (2009), Pierru (2009a), and Lobe (2009) Keef et al. (2012) support the idea of constant WACC over time and base their argumentation on the following relationship between the cost of levered equity r_e^L, cost of unlevered equity r_e^U, cost of debt r_b as well as leverage L

$$r_e^L = r_e^U + (r_e^U - r_b) \times L. \tag{1}$$

This relationship is derived from the combination of two theorems that we discuss separately in the following.

1 The first theorem necessary to derive Eq. (1) is an adjustment formula that appropriately describes the relationship between weighted average cost of capital WACC and cost of unlevered equity r_e^U. In this respect Keef et al. (2012) refer to the seminal works of Modigliani and Miller (1963) as well as Miles and Ezzell (1980) from which such adjustment formulas can be inferred. In their most general form they read as follows:

$$WACC = (1 - \tau \times l_0) \times r_e^U \tag{2a}$$

$$WACC_t = (1 + r_{e,t}^U)\left(1 - \frac{\tau \times r_f}{1 + r_f} \times l_t\right) - 1 \tag{2b}$$

They are known as the Modigliani–Miller adjustment formula (2a) and the Miles–Ezzell adjustment formula (2b), respectively. In these equations l denotes the leverage ratio whereas L in Eq. (1) stands for the debt equity ratio. Both measures of leverage can easily be converted into one another. For the risk-free rate we use the symbol r_f. Obviously, if we assume the absence of taxes, $\tau = 0$, both equations coincide and arrive at the result that WACC equals the cost of unlevered equity. Nevertheless, it is crucial to distinguish clearly between both formulas because they are based on different assumptions and only in case of the Miles–Ezzell adjustment formula (2b) do the cost of capital and the leverage ratio carry time subscripts and are thus time-dependent. In particular, the Modigliani–Miller adjustment formula (2a) can only be derived if one assumes that the amount of debt D_t does not change over time (the firm never redeems its debt) and that the firm's time horizon is either infinite or just one period. Both assumptions are clearly not met in the example at the center of the debate (see Table 1). By contrast, the Miles–Ezzell adjustment formula assumes deterministic but not necessarily constant leverage ratios[6] as well as deterministic but not necessarily constant cost of capital. Thus, the Miles-Ezzell adjustment formula is applicable to the example in Table 1 without contradiction. In a world without taxes, $\tau = 0$, it simplifies to

$$WACC_t = r_{e,t}^U. \tag{3}$$

Obviously, weighted average costs of capital can indeed be time-dependent – a fact that is neglected by Keef et al.

2 In order to derive Eq. (1), one also has to use a formula that determines WACC as the average of the cost of levered equity and debt weighted by equity and debt ratio, respectively,

$$WACC_t = (1 - l_t) \times r_{e,t}^L + l_t \times (1 - \tau) \times r_{b,t}. \tag{4}$$

Eq. (4) can be found in almost every finance textbook and is thus known as the textbook formula. In order to apply this formula significantly fewer assumptions are involved compared to the adjustment formulas. Rather, it is possible to show that the textbook formula is a trivial conclusion of the cost of capital definition

constant WACC it is indeed indispensable for the capital budgeting decision in $t = 0$ to calculate $WACC_t$ for every future period $t = 0, 1, \ldots, n - 1$ for which future costs of equity, costs of debt and future debt ratios apply. Similarly, Pierru (2009b) argues that Miller's argumentation is unsubstantiated and that the resulting discounting procedure might violate essential consistency properties.

[4] Keef et al. (2012), p. 441.

[5] Keef et al. (2012), p. 441.

[6] Whilst Miles and Ezzell derive their formula only for the case of a constant leverage ratio, it has been proven that the assumption of a deterministic but time-variant leverage ratio is sufficient to derive the formula (2b). See, e.g., (Kruschwitz & Löffler, 2006, Section 2.4.4). In this respect (Pierru, 2009a, p. 1220) is mistaken in believing that "a constant WACC implicitly requires the debt ratio to also remain constant".