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Asymmetry: Resurrecting the roots

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ABSTRACT

This note provides for a didactic survey on a range of primary methods for dealing with price asymmetry. Using Wolfram's (1971) stylized example, we argue that asymmetry can be captured in a straightforward and highly intuitive manner with first differences. While this asymmetry definition is more readily interpretable than the alternatives proposed by Wolffram (1971) and Houck (1977), we demonstrate that, theoretically, all three of these definitions are equivalent. Using data on U.S. coffee consumption, however, we illustrate that, in practice, these approaches may yield divergent conclusions on asymmetry. In such situations, the asymmetry concept based on first differences is advantageous.

1. Introduction

The estimation of so-called irreversible supply and demand functions that allow for asymmetric price responses has been a subject of ongoing research across a range of fields, including agriculture (Mundlak & Larson, 1992; Rajcaniova & Pokrivcak, 2013; Traill, Colman, & Young, 1978) and energy economics (Adeyemi & Hunt, 2014; Bachmeier & Griffin, 2003; Cologni & Manera, 2009; Frondel & Vance, 2013; Griffin & Schulman, 2005; Peltzman, 2000). While theoretical arguments in favor of asymmetric responses to rising or falling agricultural input prices were advanced by Johnson (1958), the empirical work on the topic was pushed with an analysis of aggregate farm output by Tweeten and Quance (1969a, 1969b). Their approach, which employs dummy variables that split up the price variable into two complementing explanatory terms capturing either increasing or decreasing input prices, is criticized by Wolfram (1971: 356).

Wolffram (1971) proposes an alternative technique based on cumulated price differences that, in their reply to his criticism, Tweeten and Quance (1971: 359) concede is superior to their approach, even though the application of the technique to their own data suggests otherwise (Tweeten & Quance, 1971: 360). In the aftermath of this exchange, Wolffram's technique, henceforth called the W technique, became the most popular method of partitioning an explanatory variable to allow for the estimation of a non-reversible function (Traill et al., 1978: 528), and has since served as a foundation for more sophisticated approaches, such as error-correction models (for helpful surveys, see Frey & Manera, 2007; Meyer & von Cramon-Taubadel, 2004). Despite Wolffram's (1971) and Tweeten and Quance's (1971) common belief of the superiority of the W technique, however, a number of articles have pointed to several weaknesses in its application, including the high dependence on the starting point of the data (Griffin & Schulman, 2005: 7) and its proneness to multi-collinearity problems (Saylor, 1974).

Using Wolffram's (1971) example originally conceived to demonstrate the superiority of his method over the Tweeten and Quance – henceforth TQ – approach, this note provides for a didactic survey on early asymmetry test approaches and argues that the notion of asymmetry can be captured in a straightforward and highly intuitive manner in terms of first differences. We prove that, in a deterministic context without stochastic influences, this asymmetry definition is equivalent to both Wolffram's and Houck's (1977) alternatives. Using an empirical example originating from the U.S. coffee market, however, we demonstrate that, in practice, these approaches may yield divergent conclusions with respect to asymmetry. We argue that in such situations the asymmetry concept based on first differences is advantageous for many reasons.

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Table 1
Wolffram's original example and its modification.

Original values					W technique				TQ technique				Modified y
y	x	a	Δy	Δx	w^+	w^-	Δw^+	Δw^-	x^+	x^-	Δx^+	Δx^-	$\tilde{y}$
20	10	−30	–	–	10	10	–	–	10	0	–	–	20
35	13	−30	15	3	13	10	3	0	13	0	3	0	35
29	11	−4	−6	−2	13	12	0	2	0	11	−13	11	13
44	14	−26	15	3	16	12	3	0	14	0	14	−11	40
59	17	−26	15	3	19	12	3	0	17	0	3	0	55
44	12	8	−15	−5	19	17	0	5	0	12	−17	12	16
35	9	8	−9	−3	19	20	0	3	0	9	0	−3	7
70	16	−10	35	7	26	20	7	0	16	0	16	−9	50
90	20	−10	20	4	30	20	4	0	20	0	4	0	70
84	18	30	−6	−2	30	22	0	2	0	18	−20	18	34

2. A reassessment of Wolffram's example

Wolffram (1971: 357) criticizes that any irreversible relationship $y = f(x)$ between a dependent variable y and an explanatory variable x cannot be determined exactly with the TQ approach, which splits x into two complementary variables, x^+ and x^-. Variable x^+ is defined as $x_1^+ := x_1$ and for $i > 1$ by

$$x_i^+ := x_i, \quad \text{if} \quad x_i > x_{i-1}, \tag{1}$$

and $x_i^+ := 0$ otherwise, where subscript i is used to denote the observation, while x^- is defined in a similar way: $x_1^- := 0$, and for $i > 1$:

$$x_i^- := x_i, \quad \text{if} \quad x_i \le x_{i-1}, \tag{2}$$

and $x_i^- := 0$ otherwise. By definition, $x_i^+ + x_i^- = x_i$ for all i.

As an alternative to the TQ decomposition of x, Wolffram (1971) suggests taking cumulated increases and decreases of the explanatory variable x, denoted here by w_i^+ and w_i^-, respectively. In detail, Wolffram (2000: 351–352) defines his approach by $w_1^+ = w_1^- := x_1$ and for $i > 1$,

$$w_i^+ := w_{i-1}^+ + D_i^+ \cdot (x_i - x_{i-1}) = w_1^+ + \sum_{k=2}^{i} (x_k - x_{k-1}) D_k^+, \tag{3}$$

$$w_i^- := w_{i-1}^- - D_i^- \cdot (x_i - x_{i-1}) = w_1^- - \sum_{k=2}^{i} (x_k - x_{k-1}) D_k^-, \tag{4}$$

where $D_k^+ := 1$ for $x_k > x_{k-1}$ and 0 otherwise, while $D_k^- := 1 - D_k^+$.[1] From this definition, it becomes obvious that w^+ and w^- include cumulated price in- and decreases, respectively.

To demonstrate the superiority of his approach over the TQ decomposition, Wolffram (1971) conceives a straightforward example presented in Table 1. For this purpose, Wolffram (1971: 358) assumes the following exact relationship between the predefined values of dependent variable y and those of the explanatory variable x, which is split up into x^+ and x^- according to the TQ decomposition:

$$y_i = a_i + 5x_i^+ + 3x_i^-. \tag{5}$$

In this equation, potential residual terms u_i are set to zero: $u_i = 0$, thereby attributing the varying differences between the predefined values y_i and the predicted values $\hat{y}_i := 5x_i^+ + 3x_i^-$ to variable a, whose components are also shown in Table 1.

As Wolffram (1971: 357) emphasizes, this contrasts with the classical Ordinary Least Squares (OLS) framework, in which variable a would adopt the role of a constant: $a = a_0$. It is not surprising, therefore, that when applying OLS methods, one obtains the following estimation equation for which both coefficient estimates, 6.25 and 6.99, differ greatly from the predefined coefficients in Eq. (5)[2]:

$$y_i = -40.23(11.03) + 6.25(0.74)x_i^+ + 6.99(0.88)x_i^- + \hat{u}_i, \tag{6}$$

with $R^2 = 0.912$, $\hat{u}_i \neq 0$ for all i, and standard errors reported in parentheses. In contrast, Wolffram shows that the correct coefficients 5 and 3 are reproduced – apart from the sign of coefficient 3 – by using the proposed W technique and regressing y on w^+ and w^-:

$$y_i = 0 + 5w_i^+ - 3w_i^-, \tag{7}$$

where $\hat{u}_i = 0$ for all i and, hence, $R^2 = 1$. Obviously, this example was constructed in such a way that precisely this result will be obtained when using the W technique.

In what follows, we demonstrate that Wolffram's critique with regard to the TQ decomposition is generally correct, although it is inappropriate to blame the TQ decomposition for a poor performance in this specific example. The reason is that the differences between the coefficient estimates reported in Eq. (6) and the true coefficients of 5 and 3 is merely the result of the fact that the varying values a_i are approximated by a constant when Eq. (5) is estimated by OLS. If one estimates Eq. (5) by employing variable a as an additional regressor, thereby avoiding any omitted-variable bias, one can exactly reproduce the coefficients given in Eq. (5).

Furthermore, one point that immediately emerges from Wolffram's example is that in case of irreversibility, one may expect distinct intercepts a^+ and a^-, $a^+ \neq a^-$, as is shown in the following modification of Wolffram's example:

$$\tilde{y}_i = -30D_i^+ - 20D_i^- + 5x_i^+ + 3x_i^-, \tag{8}$$

with $a^+ = -30D_i^+$ and $a^- = -20D_i^-$ and the modified values $\tilde{y}_i$ for the dependent variable being shown in Table 1. Eq. (8) reflects the fact that in case of asymmetry, one would expect two entirely distinct functions, one for each of the two different regimes of either increasing or decreasing values of x.

If one falsely estimates Eq. (8) by using a common intercept, the following OLS results are obtained:

$$\tilde{y}_i = -24.87(1.89) + 4.67(0.13)x_i^+ + 3.36(0.15)x_i^-. \tag{9}$$

In statistical terms, the coefficient estimates of x_i^+ and x_i^- are significantly different from the true vales 5 and 3, respectively. Clearly, these estimation results, which seem to support Wolffram's criticism with respect to the TQ decomposition, are due to omitted-variable bias. This bias could be readily avoided by including two dummy variables that capture the different intercepts, rather than

[1] Using the dummy variables D_i^+ and D_i^-, the TQ decomposition can be concisely described by $x_i^+ = D_i^+ x_i$ and $x_i^- = D_i^- x_i$ for $i > 1$ (Meyer & von Cramon-Taubadel, 2004: 594).

[2] Wolffram (1971: 358) reported an estimate of −43.16 for the constant, which appears to be wrong.