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Spatial autoregressive models with unknown heteroskedasticity: A comparison of Bayesian and robust GMM approach ☆

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ABSTRACT

Most of the estimators suggested for the estimation of spatial autoregressive models are generally inconsistent in the presence of an unknown form of heteroskedasticity in the disturbance term. The estimators formulated from the generalized method of moments (GMM) and the Bayesian Markov Chain Monte Carlo (MCMC) frameworks can be robust to unknown forms of heteroskedasticity. In this study, the finite sample properties of the robust GMM estimator are compared with the estimators based on the Bayesian MCMC approach for the spatial autoregressive models with heteroskedasticity of an unknown form. A Monte Carlo simulation study provides evaluation of the performance of the heteroskedasticity robust estimators. Our results indicate that the MLE and the Bayesian estimators impose relatively greater bias on the spatial autoregressive parameter when there is negative spatial dependence in the model. In terms of finite sample efficiency, the Bayesian estimators perform better than the robust GMM estimator. In addition, two empirical applications are provided to evaluate relative performance of heteroskedasticity robust estimators.

1. Introduction

Most of the estimation methods suggested in the literature are valid under the assumption that the disturbance terms of spatial models are independently and identically distributed (i.i.d.) or i.i.d. normal. However, in many regression applications, heteroskedasticity may well be present. For example, cross-sectional units usually differ in size and some other characteristics, which in turn implies that the disturbance terms in the regression analyses across these cross-sectional units may be heteroskedastic. It may also be present in a random coefficient model, where the parameters of the model are random around fixed values. In this case, heteroskedasticity depends on the exogenous variables of the spatial models. Moreover, in regression analysis, many dependent variables are constructed by data aggregation. In such a case, heteroskedasticity arises from the process of averaging with different numbers of observations when the data is getting aggregated (Griffiths, 2007; Lee, 2010).[1]

In the present study, we evaluate the performance of various heteroskedasticity robust estimators suggested in the literature for spatial autoregressive models. To this end, we conduct a Monte Carlo study and provide two empirical illustrations to show how these estimators perform in applied research.

In the presence of heteroskedastic disturbances, the ML and GMM estimators are generally inconsistent. The ML estimator is inconsistent if heteroskedasticity is not incorporated into estimation, because the likelihood function is not maximized at the true parameter values.[2] The GMM estimators are also inconsistent since the moment functions are often designed under the assumption that disturbances are i.i.d. (Kelejian and Prucha, 1998, 1999; Liu et al., 2010). Hence, the orthogonality conditions for the moment functions may not be satisfied.

To handle unknown forms of heteroskedasticity, Kelejian and Prucha (2010) extend their two-step GMM estimation approach by modifying the moment functions for a spatial model that has a spatial lag in the

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[1] For some recent empirical studies, see Lin and Lee (2010) and Doğan and Taşpınar (2013b).

[2] For many spatial model specifications, the ML estimation has been the most widely used technique and has often been the only technique that is implemented. The ML approach is well treated in Anselin (1988), Lee (2004) and LeSage and Pace (2009).

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dependent variable and the disturbance term (for short SARAR(1,1)). Badinger and Egger (2011) extend the robust estimation approach in Kelejian and Prucha (2010) to the case of SARAR(p,q) specification. Lin and Lee (2010) suggest a one-step robust GMM estimator for the model with only spatial dependence in the dependent variable (for short SARAR(1,0)).[3] In this approach, the parameters of a spatial model are simultaneously estimated by a GMM estimator formulated from the combination of a set of linear and quadratic moment functions (Lee, 2007a, 2007b; Lee and Liu, 2010; Liu et al., 2010).

The moment functions considered for both two-step and one-step GMM estimators are motivated by the reduced form of the spatial models. For example, the reduced form for the case of SARAR(1,0) indicates that the endogenous spatial lag of the dependent variable is a function of a stochastic and a non-stochastic variable. The linear moment functions are formulated from the non-stochastic part and the quadratic moment functions are formulated for the stochastic part. The two-step GMM approach of Kelejian and Prucha is motivated by computational simplicity as the ML estimation involves significant computational burden for the large samples. Despite the computational advantage, the two step GMM estimator is inefficient relative to the one-step GMM estimators suggested in Lee (2007a, 2007b), Liu et al. (2010) and Lee and Liu (2010).

An alternative to ML and GMM/IV estimation methods is the Bayesian estimation method, which has been receiving attention in recent years. Bayesian data analysis is distinctly different from the classical (or frequentist) analysis in its treatment of parameters of the model. In Bayesian econometrics the parameter vector is a random variable, and Bayesian analysts formulate probabilistic statements about the parameters before observing any data. These ex-ante probabilities are called priors and usually take the form of a probability distribution with known moments. This notion of the prior probabilities (or subjective probabilities) is totally absent in classical econometrics where all estimation and inference are based on observed data. In both approaches, the likelihood function has the same functional form and reflects the relation between data and the parameters of interest. In Bayesian approach, the likelihood function is combined with prior functions via Bayes' rule to construct the posterior probability distribution of parameters. The posterior distribution function of the parameter vector contains the information necessary for the estimation and inference.

The Bayesian approach requires the evaluation of higher dimensional integrals to obtain posterior expectations, marginal likelihoods, and predictive densities. The application of the Bayesian methods to the estimation of spatial models follows the path of the progress that has been made in the context of Bayesian computation techniques. Hepple (1995b) identifies four phases for the development of Bayesian computation techniques. In the first phase, Bayesian studies involved problems that can be characterized with well known probability distributions such that the characteristics of posterior distributions such as means and covariances can be analytically derived. In the second phase, Bayesian studies focused on techniques through which problems involving multidimensional probability distributions can be reduced to univariate or bivariate integrations. In this phase, numerical techniques for the univariate and bivariate integration were used. In the third phase, Bayesian analysts worked on efficient procedures for higher order integrations. Gauss–Hermite, importance sampling and Monte Carlo integration techniques were used to tackle complicated and high-dimensional problems. In the fourth phase, Markov Chain Monte Carlo (MCMC) simulation techniques were introduced that make the computation of higher dimension problems feasible. The advent of the MCMC approach represents a shift in thinking, where the focus on the question of analytical moment calculation is replaced with a more general question of sampling issues from the posterior distributions (Albert and Chib, 1993; Casella and George, 1992; Chib, 2001). In the Gibbs sampling version of the MCMC approach, the joint posterior distribution is decomposed into conditional posterior distributions through which random draws (or a simulated sample) can be obtained. Inferences such as posterior mean and posterior covariance matrix can be estimated from the simulated sample obtained from the conditional posterior densities.

The development in Bayesian computation techniques provides a wide range of tools that can be applied to the estimation of spatial models. The early literature on the Bayesian perspective on spatial models uses combinations of tools developed during the period from the first to the third phase. For example, Hepple (1979) analytically derives the joint posterior and marginal posterior distributions of parameters for a spatial model containing a spatial lag in the disturbance term. The posterior moments are calculated through numerical univariate and bivariate integration techniques.

Anselin (1982, 1988) considers the Bayesian approach for pure spatial autoregressive and spatial error models. Diffuse priors for parameters of models are suggested and marginal posterior distributions of parameters are analytically derived. The posterior mean of the autoregressive parameter for a pure spatial autoregressive model in Anselin (1982) is estimated with univariate numerical integration. A small Monte Carlo simulation study in Anselin (1982) demonstrates that the Bayesian estimator performs as well as the ML estimator for larger values of the autoregressive parameter and larger samples.[4]

Hepple (1995a, 1995b) develops Bayesian analyses for major spatial specifications including the SARAR(1,0) model, SARAR(0,1) (or the SEM) model and spatial moving average models (for short SARMA(0,1)). In each case, the joint posterior distributions of the parameters are stated from which the marginal posterior of the spatial autoregressive parameters is analytically derived. The analytical derivation for the marginal posterior distribution of the parameters of the exogenous variables is not available as the spatial autoregressive parameters can not be analytically integrated out from the joint posterior distribution. However, the dimension of joint posterior can be reduced to two dimensions so that bivariate numerical integration techniques can be used for the estimation of the marginal posterior moments. As a result, the estimate of the spatial autoregressive parameters can be obtained through univariate numerical integration, and the estimates of the parameters of the exogenous variables can be obtained through bivariate numerical integration over a grid of pairs of values.[5]

Hepple (2002, 2003) provides further analytical simplifications such that the moments of the marginal posterior distributions of the exogenous variables in spatial models with only one autoregressive parameter can be obtained through univariate numerical integration. However, for the case of SARAR(1,1) and SARMA(1,1) where there are two spatial autoregressive parameters the calculation of these moments again requires bivariate integration over the parameter space of autoregressive parameters.

The recent studies use the MCMC approach to estimate spatial models. This approach is more appropriate for cases where marginal posterior distributions are difficult to simplify analytically and to integrate numerically. The MCMC approach is introduced for most types of spatial models in LeSage (1997) and LeSage and Pace (2009). Kakamu and Wago (2008) compare finite sample properties of the Bayesian estimators based on the MCMC approach with that of the ML estimator for the static panel spatial autoregressive model. The Monte Carlo simulation results in Kakamu and Wago (2008) show that the Bayesian estimator is virtually as efficient as the ML estimator.

In spatial models, the boundaries of the parameter space for spatial autoregressive parameters are known, which facilitate the

[3] For a robust 2SLS estimator of the SARAR(1,0) specification, see Anselin (2006).

[4] To the best of our knowledge, Anselin (1982) is the first study, where small sample properties of the ML estimator are compared with the frequentist properties of the Bayesian estimator in the context of spatial models.

[5] Note that as the dimension of spatial parameter increases, the dimension of numerical integration rises. For example, for the case of SARAR(1,1) the marginal posterior of autoregressive parameters is two dimensional.