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A wavelet-based assessment of market risk: The emerging markets case☆

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ABSTRACT

The measurement of market risk poses major challenges to researchers and different economic agents. On one hand, it is by now widely recognized that risk varies over time. On the other hand, the risk profile of an investor, in terms of investment horizon, makes it crucial to also assess risk at the frequency level. We propose a novel approach to measuring market risk based on the continuous wavelet transform. Risk is allowed to vary both through time and at the frequency level within a unified framework. In particular, we derive the wavelet counterparts of well-known measures of risk. One is thereby able to assess total risk, systematic risk and the importance of systematic risk to total risk in the time-frequency space. To illustrate the method we consider the emerging markets case over the last twenty years, finding noteworthy heterogeneity across frequencies and over time, which highlights the usefulness of the wavelet approach.

1. Introduction

The assessment of market risk has long posed a challenge to many types of economic agents and researchers (see, for instance, Granger, 2002 for an overview). Market risk arises from the random unanticipated changes in the prices of financial assets and measuring it is crucial for investors. Besides its interest to portfolio managers, the assessment of market risk is relevant for the overall risk management in banks and bank supervisors. Although bank failures are traditionally related with an excess of non-performing loans (the so-called credit risk), the failure of the Barings Bank in 1995 showed how market risk can lead to bankruptcy. Furthermore, market risk has received increasing attention in recent years as banks' financial trading activities have grown.

Although the measurement of market risk has a long tradition in finance, there is still no universally agreed upon definition of risk. The modern theory of portfolio analysis dates back to the pioneering work of Harry Markowitz in the 1950s. The starting point of portfolio theory rests on the assumption that investors choose between portfolios on the basis of their expected return, on the one hand, and the variance of their return, on the other. The investor should choose a portfolio that maximizes expected return for any given variance, or alternatively, minimizes variance for any given expected return. The portfolio choice is determined by the investor's preferred trade-off between expected return and risk. Hence, in his seminal paper, Markowitz (1952) implicitly provided a mathematical definition of risk, that is, the variance of returns. In this way, risk is thought in terms of how spread-out the distribution of returns is.

Later on, the Capital Asset Pricing Model (CAPM) emerged through the contributions of Sharpe (1964) and Lintner (1965a, 1965b). According to the CAPM, the relevant risk measure in holding a given asset is the systematic risk, since all other risks can be diversified away through portfolio diversification. The systematic risk, measured by the beta coefficient, is a widely used measure of risk. In statistical terms, it is assumed that the variability in each stock's return is a linear function of the return on some larger market with the beta reflecting the responsiveness of an asset to movements in the market portfolio. For instance, in the context of international portfolio diversification, the country risk is defined as the sensitivity of the country return to a world stock return. Traditionally, it is assumed that beta is constant through time. However, empirical research has found evidence that betas are time varying (see, for example, the pioneer work of Blume, 1971, 1975). Such a finding led to a surge in contributions to the literature (see, for example, Alexander & Benson, 1982; Collins, Ledolter, & Rayburn, 1987; Fabozzi & Francis, 1977, 1978; Ferson & Harvey, 1991, 1993;

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Ghysels, 1998; Harvey, 1989, 1991; Sunder, 1980, among others). One natural implication of such a result is that risk measurement must be able to account for this time-varying feature.

Besides the time-variation, risk management should also take into account the distinction between the short and long-term investor (see, for example, Candelon, Piplack, & Straetmans, 2008). In fact, the first kind of investor is naturally more interested in risk assessment at higher frequencies, that is, short-term fluctuations, whereas the latter focuses on risk at lower frequencies, that is, long-term fluctuations. Analysis at the frequency level provides a valuable source of information, considering that different financial decisions occur at different frequencies. Hence, one has to resort to the frequency domain analysis to obtain insights into risk at the frequency level.

In this paper, we re-examine risk measurement through a novel approach, wavelet analysis. Wavelet analysis constitutes a very promising tool as it represents a refinement in terms of analysis in the sense that both time and frequency domains are taken into account. In particular, one can resort to wavelet analysis to provide a unified framework to measure risk in the time-frequency space. As both time and frequency domains are encompassed, one is able to capture the time-varying feature of risk while disentangling its behavior at the frequency level. In this way, one can simultaneously measure the evolving risk exposure and distinguish the risk faced by short and long-term investors. Although wavelets have been more popular in fields such as signal and image processing, meteorology, and physics, among others, such analysis can also shed fruitful light on several economic phenomena (see, for example, the pioneering work of Ramsey & Lampart, 1998a, 1998b; Ramsey & Zhang, 1996, 1997). Recent work using wavelets includes that of, for example, Kim and In (2003, 2005), who investigate the relationship between financial variables and industrial production and between stock returns and inflation, Gençay, Selçuk, and Whitcher (2005), Gençay, Whitcher, and Selçuk (2003) and Fernandez (2005, 2006), who study the CAPM at different frequency scales, Connor and Rossiter (2005) focus on commodity prices, In and Kim (2006) examine the relationship between the stock and futures markets, Gallegati and Gallegati (2007) provide a wavelet variance analysis of output in G-7 countries, Gallegati, Palestrini, and Petrini (2008) and Yogo (2008) resort to wavelets for business cycle analysis, Rua (2011) focuses on forecasting GDP growth in the major euro area countries, and others (see Crowley, 2007, for a survey). However, up to now, most of the work drawing on wavelets has been based on the discrete wavelet transform. In this paper we focus on the continuous wavelet transform to assess market risk (see also, for example, Aguiar-Conraria & Soares, 2011a, 2011b, 2011c; Crowley & Mayes, 2008; Raihan, Wen, & Zeng, 2005; Rua, 2010, 2012; Rua & Nunes, 2009; Tonn, Li, & McCarthy, 2010).

We provide an illustration by considering the emerging markets case. The new equity markets that have emerged around the world have received considerable attention in the last two decades, leading to extensive recent literature on this topic (see, for example, Bekaert & Harvey, 1995, 1997, 2000, 2002, 2003; Chambet & Gibson, 2008; De Jong & De Roon, 2005; Dimitrakopoulos, Kavussanos, & Spyrou, 2010; Estrada, 2000; Garcia & Ghysels, 1998; Harvey, 1995, among others). The fact that the volatility of stock prices changes over time has long been known (see, for example, Fama, 1965), and such features have also been documented for the emerging markets. The time variation of risk comes even more naturally in these countries due to the changing economic environment resulting from capital market liberalizations or the increasing integration with world markets and the evolution of political risks. In fact, several papers have acknowledged time varying volatility and betas for the emerging markets (see, for example, Bekaert & Harvey, 1997, 2000, 2002, 2003; Estrada, 2000; Santis & Imrohoroglu, 1997). Moreover, the process of market integration is a gradual one, as emphasized by Bekaert and Harvey (2002). Therefore, methods that allow for gradual transitions at changing speeds, such as wavelets, are preferable to segmenting the analysis into various subperiods. Hence, the emerging markets case makes an interesting example for measuring risk with the continuous wavelet transform.

This paper is organized as follows. In Section 2, the main building blocks of wavelet analysis are presented. In Section 3, we provide the wavelet counterpart of well-known risk measures. In Section 4, an application to the emerging markets case is provided. Section 5 concludes.

2. Wavelet analysis

The wavelet transform decomposes a time series in terms of some elementary functions, the daughter wavelets or simply wavelets $\psi_{\tau,s}(t)$. Wavelets are "small waves" that grow and decay in a limited time period. These wavelets result from a mother wavelet $\psi(t)$ that can be expressed as a function of the time position τ (translation parameter) and the scale s (dilation parameter), which is related with the frequency. While the Fourier transform decomposes the time series into infinite length sines and cosines (see, for example, Priestley, 1981), discarding all time-localization information, the basis functions of the wavelet transform are shifted and scaled versions of the time-localized mother wavelet. In fact, wavelet analysis can be seen as a refinement of Fourier analysis. More explicitly, wavelets are defined as

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right) \tag{1}$$

where $(1/\sqrt{s})$ is a normalization factor to ensure that wavelet transforms are comparable across scales and time series. To be a mother wavelet, $\psi(t)$ must meet several conditions (see, for example, Percival & Walden, 2000): it must have zero mean, $\int_{-\infty}^{+\infty}\psi(t)dt = 0$; its square integrates to unity, $\int_{-\infty}^{+\infty}\psi^2(t)dt = 1$, which means that $\psi(t)$ is limited to an interval of time; and it should also satisfy the so-called admissibility condition, $0 < C_\psi = \int_0^{+\infty}(|\widehat{\psi}(\omega)|^2)/(\omega)d\omega < +\infty$ where $\widehat{\psi}(\omega)$ is the Fourier transform of $\psi(t)$, that is, $\widehat{\psi}(\omega) = \int_{-\infty}^{+\infty}\psi(t)e^{-i\omega\tau}dt$. The last condition allows the reconstruction of a time series $x(t)$ from its continuous wavelet transform, $W_x(\tau, s)$. Thus, it is possible to recover $x(t)$ from its wavelet transform through the following formula

$$x(t) = \frac{1}{C_\psi}\int_0^{+\infty}\left[\int_{-\infty}^{+\infty}\frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right)W_x(\tau,s)d\tau\right]\frac{ds}{s^2} \tag{2}$$

The continuous wavelet transform of a time series $x(t)$ with respect to $\psi(t)$ is given by the following convolution

$$W_x(\tau,s) = \int_{-\infty}^{+\infty}x(t)\psi^*_{\tau,s}(t)dt = \frac{1}{\sqrt{s}}\int_{-\infty}^{+\infty}x(t)\psi^*\left(\frac{t-\tau}{s}\right)dt \tag{3}$$

where * denotes the complex conjugate. For a discrete time series, $x(t)$, $t = 1, \ldots, N$ we have

$$W_x(\tau,s) = \frac{1}{\sqrt{s}}\sum_{t=1}^{N}x(t)\psi^*\left(\frac{t-\tau}{s}\right) \tag{4}$$

Although it is possible to compute the wavelet transform in the time domain using Eq. (4), a more convenient way to implement it is to carry out the wavelet transform in Fourier space (see, for