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A two-component equilibrium-diffusion limit

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This paper is dedicated to Prof. Edward Larsen on the occasion of his 60th birthday. One of us (JEM) has collaborated with Ed for over 20 years, while the other (JDD) recently received his Ph.D. from the University of Michigan under Ed's, direction. Both of us learned asymptotics from Ed, so the subject of this paper is truly appropriate. We congratulate Prof. Larsen on his extraordinary record of outstanding technical achievement, and look forward to many more years of fruitful collaborations.

Abstract

The equilibrium-diffusion limit of the radiative transfer equations is characterized by a medium that is optically thick and diffusive for photons of all frequencies. In reality, this condition is almost never met because the transport medium tends to be optically thin for photons of sufficiently high frequency. Motivated by this fact, we derive a new asymptotic limit of the radiative transfer equations that is characterized by two photon components: one for which the medium is optically thick and diffusive, and the other for which the medium is optically thin. In this limit, the leading-order material temperature satisfies a time-dependent diffusion equation, and the leading-order radiation intensity for the optically thick photons is given by the Planck function evaluated at the leading-order material temperature, but the radiation intensity for the optically thin photons is zero through first order. The $O(\epsilon^2)$ radiation intensity for the optically thin photons satisfies a quasi steady-state transport equation with zero interaction terms and a Planck emission term that depends upon the leading-order material temperature. We also discuss alternative scalings associated with the two-component limit that are characterized by a stronger coupling between the material and the optically thin component.

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1. Introduction

The equilibrium-diffusion limit (Pomraning, 1973; Larsen et al., 1983) is an asymptotic limit of the radiative transfer equations in which the transport medium is assumed to be optically thick to photons of all frequencies. In reality a diffusive, transport medium is essentially never optically thick to all photon frequencies. Rather, it is often optically thin to the photons with frequencies much greater than the average frequency of the local Planck distribution, and optically thick to the remainder of the photons in the distribution. In this paper, we introduce an asymptotic limit of the radiative transfer equations characterized by a two-component system of photons: one for which the medium is optically thick and diffusive, and the other for which the medium is optically thin. We also discuss alternative scalings associated with the two-component limit that are characterized by a stronger coupling between the material and the optically thin component. Finally, we give a brief summary and discuss future work.

2. The two-component equations

The radiative transfer equations for the two-component system are

$$\frac{1}{c}\frac{\partial\psi_1}{\partial t}+\vec{\Omega}\cdot\vec{\nabla}\psi_1+\sigma_{t,1}\psi_1=\frac{1}{4\pi}\sigma_{s,1}\phi_1+(\sigma_{t,1}-\sigma_{s,1})B_1, \tag{1}$$

$$\frac{1}{c}\frac{\partial\psi_2}{\partial t}+\vec{\Omega}\cdot\vec{\nabla}\psi_2+\sigma_{t,2}\psi_2=\frac{1}{4\pi}\sigma_{s,2}\phi_2+(\sigma_{t,2}-\sigma_{s,2})B_2, \tag{2}$$

$$C_v\frac{\partial T}{\partial t}=\int_{\Gamma_1}(\sigma_{t,1}-\sigma_{s,1})(\phi_1-4\pi B_1)\,\mathrm{d}\nu+\int_{\Gamma_2}(\sigma_{t,2}-\sigma_{s,2})(\phi_2-4\pi B_2)\,\mathrm{d}\nu, \tag{3}$$

where

$$\phi_i=\int_{4\pi}\psi_i\,\mathrm{d}\Omega,\quad i=1,2, \tag{4}$$

where the i is the photon component index, $i = 1$ refers to the optically thick component, $i = 2$ refers to the optically thin photon component, $\psi_i(t,\vec{r},\vec{\Omega},\nu)$ is the radiation intensity for the ith photon component, $\sigma_{t,i}$ is the macroscopic total cross-section for the ith photon component, $\sigma_{s,i}$ is the macroscopic scattering cross-section for the ith photon component, B_i is the Planck function for the ith photon component, $T(t,\vec{r})$ is the material temperature, c is the speed of light, t is the time variable, $\vec{\nabla}$ is the spatial gradient, $\vec{\Omega}$ is the photon direction variable, ν is the photon frequency variable, Γ_i is the frequency domain associated with the ith photon component, and C_v is the material heat capacity. The quantity $\sigma_{t,i}-\sigma_{s,i}$ represents the macroscopic absorption cross-section for the ith component because the total cross-section is the sum of the absorption and scattering cross-sections. All material properties are assumed to depend upon both space and temperature. The cross-sections are also assumed