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Journal of Crystal Growth 283 (2005) 15–30

JOURNAL OF CRYSTAL GROWTH

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Modelling the effects of the pressure changes in the case of the growth of a thin sheet in an edge-defined film-fed growth (EFG) System

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Received 7 October 2004; received in revised form 16 May 2005; accepted 16 May 2005
Available online 18 July 2005
Communicated by T. Hibiya

Abstract

In this paper it is shown that due to the pressure p in the furnace there are three limitations $\underline{\alpha_c}$, $\overline{\alpha_c}$, $\overline{\overline{\alpha_c}}$ for the contact angle α_c. The lower limit $\underline{\alpha_c}$ is in the range $(0, \pi/2 - \alpha_e)$, depends on p and on the half-thickness w of the die. The upper limit $\overline{\alpha_c}$ is in the range $(\pi/2 - \alpha_e, \pi/2)$ and increases when p increases. The limit $\overline{\overline{\alpha_c}}$ is in the range $(\pi/2 - \alpha_e, \overline{\alpha_c})$ and depends on p and w. It is shown also, that there exist two limitations $w_1(p)$ and $w_2(p)$ for w. If w is in the range $(0, w_1(p))$, then the growth angle is achieved for the contact angle in the range $(\pi/2 - \alpha_e, \overline{\overline{\alpha_c}})$, and if $w \in (w_1(p), w_2(p))$ the growth angle is achieved for the contact angle in the range $(\pi/2 - \alpha_e, \overline{\alpha_c})$. The menisci are concave. If $w > w_2(p)$, then the growth angle is achieved for the contact angle in the range $(\underline{\alpha_c}, \overline{\alpha_c})$ and the meniscus is convex–concave. Numerical examples are given for thin silicon sheets of half-thickness $0.5(\mathrm{cm} \times 10^{-2})$.

PACS: 81.10.Fq; 68.08.Bc; 46.15.−x; 44.90.+c; 05.45.−a

Keywords: A1. Single crystal growth; A2. Growth from the melt; A2. Edge-defined film-fed growth

1. Introduction

The edge-defined film-fed growth (EFG) method is performed to achieve sheets with constant half-thickness when the pulling rate v, melt temperature at the meniscus basis T_0, pressure in the furnace p are constant and the bottom line of the melt/gas meniscus is fixed to the outer edge of the die. In reality v, T_0, p

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0022-0248/$ - see front matter
doi:10.1016/j.jcrysgro.2005.05.044

can change during the growth and for some materials the bottom line of the melt/gas meniscus on the die can move.

In the last 20 years, many experimental and theoretical studies have been reported regarding the growth process [1–44]. In this paper mainly the effects of the pressure changes are investigated in the case of the growth of a thin sheet.

2. Theoretical results

The system of differential equations which governs the evolution in time of the half-thickness $x = x(t)$ and the meniscus height $h = h(t)$ is

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = -v\ \tan[\alpha(x,h;p,w) - \alpha_1], \\ \dfrac{\mathrm{d}h}{\mathrm{d}t} = v - \dfrac{1}{\Lambda\rho_2}[\lambda_1 G_1(x,h;v,T_0) - \lambda_2 G_2(x,h;v,T_0)]. \end{cases} \tag{1}$$

For details see Fig. 1 and Refs. [19,20,32,33,39,42,43]. The significance of these quantities and their values for Si (see [38,43,45]) are given in Table 1.

The function $\alpha = \alpha(x,h;p,w)$ is obtained from the Young–Laplace equation of the capillary surface in equilibrium in the presence of pressure:

$$\left[1+\left(\frac{\partial z}{\partial y}\right)^2\right]\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x}\frac{\partial z}{\partial y}\frac{\partial^2 z}{\partial x \partial y} + \left[1+\left(\frac{\partial z}{\partial x}\right)^2\right]\frac{\partial^2 z}{\partial y^2} = \frac{\rho_1 g z - p}{\gamma}\left[1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2\right]^{3/2}. \tag{2}$$

For sheets, solutions $z = z(x)$ depending only on the coordinate x are searched, i.e. the Ox axis is normal to the sheet surface. For this type of functions Eq. (2) becomes

$$\frac{\mathrm{d}^2 z}{\mathrm{d}x^2} = \frac{\rho_1 g z - p}{\gamma}\left[1+\left(\frac{\mathrm{d}z}{\mathrm{d}x}\right)^2\right]^{3/2}. \tag{3}$$

Eq. (3) is transformed into the system

$$\begin{cases} \dfrac{\mathrm{d}z}{\mathrm{d}x} = -\tan\alpha \\ \dfrac{\mathrm{d}\alpha}{\mathrm{d}x} = -\dfrac{\rho_1 g z}{\gamma}\dfrac{1}{\cos\alpha} + \dfrac{p}{\gamma}\dfrac{1}{\cos\alpha}, \end{cases} \tag{4}$$

for which the following boundary values are considered:

$$z(w) = 0,\ \ \alpha(w) = \alpha_c;\ \ \alpha_c \in \left(0,\ \frac{\pi}{2}\right). \tag{5}$$

The solution of the boundary value problem (4)–(5) is denoted by:

$$z = z(x;\alpha_c,w,p)\ \text{ and }\ \alpha = \alpha(x;\alpha_c,w,p). \tag{6}$$

For physical reasons these functions has to be considered only for $x \in (0,\ w]$ (w—small for thin sheets) and only if the following conditions hold:

(i) $$z = z(x;\alpha_c,w,p) > 0\ \text{ for }\ x \in (0,w); \tag{7}$$